The Evolution Of A Primordial Galactic Magnetic Field[5]

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ABSTRACT

We consider the hypothesis that galactic magnetic fields are primordial. We also discuss the various objections to this hypothesis. We assume that there was a magnetic field present in the galactic plasma before the galaxy formed. After the galactic disk formed, the lines of force thread through it and remain connected to the external cosmic medium. They enter through one side of the disk, proceed horizontally a distance l in the disk, and then leave through the other side. We find that the lines of force are stretched rotation of the galactic disk, which amplifies the toroidal component of the field and increases l. When the magnetic field is strong enough, it produces ambipolar velocities that try to lift the line out of the galactic disk but in opposite directions on different parts of the line. The result is, instead of the line being expelled from the disk, its horizontal length l is shortened, both in the radial, and in the toroidal direction. This leads to a reduction of the rate of horizontal stretching, and, finally, a reduction in the magnetic field strength. After a sufficient time, the magnetic field at all points goes through this stretching and reduction, and the field strength approaches a universal function of time. This function is slowly decreasing, and only depends on the ambipolar properties of the interstellar medium. At any given time the magnetic field is toroidal, and has the same strength almost everywhere. On the other hand, it turns out that its direction varies rapidly with radius, changing sign every one hundred parsecs field dominate over that of the other. The resulting field has a net Faraday rotation. If such a field were observed with low resolution in an external gal axy then the field would appear toroidal in between the spiral arms. The spiral density wave would turn it so that the lines appear to trace out the spiral arm, although the apparent lines really are the sum of pieces of magnetic lines as they cross the disk. They do not necessarily extend very far along the arms. We contend that this model of the magnetic field, which arises naturally from a primordial origin, can fit the observations as well as other models for the magnetic field, such as those arising

from the mean field dynamo theory. Finally, because the field lines are topologically threaded through the disk they cannot be expelled from the disk. This counters the objection against the primordial origin, that such a field could not survive very long in the galaxy.

Subject headings: galaxies:magnetic fields–ISM:magnetic field–magnetohydrodynamic:MH–plasmas

1. Introduction

There are two schools of thought as to the origin of the galactic magnetic field. The first school holds that, after formation of the galactic disk, there was a very weak seed field of order 10^{-17}gauss or so, and that this field was amplified to its present strength by dynamo action driven by interstellar turbulence. The description of how this happens is well known(Ruzmaikin, Shukorov and Sokoloff 1988), and is summarized by the mean field dynamo theory.

The second school of thought concerning the origin of the galactic field is, that it is primordial. That is, it is assumed that there was a reasonably strong magnetic field present in the plasma before it collapsed to form the galaxy. It is assumed that the field was coherent in direction and magnitude prior to galactic formation. The actual origin of the magnetic field is not specified. However, it could be formed by dynamo action during the actual formation of the galaxy, (Kulsrud et al 1996).

This paper discusses the evolution of the structure and strength of such a primordial magnetic field during the life of the galaxy. The origin of the field prior to formation of the galaxy is not considered.

It was Fermi(1949) who first proposed that the galactic magnetic field was of primordial origin. Piddington(1957, 1964, 1967, 1981) in a number of papers suggested how this might actually have happened. However, he supposed the magnetic field strong enough to influence the collapse. One of the first problems with the primordial origin is the wrap up problem. It was pointed out by Hoyle and Ireland(1960, 1961) and by $\hat{O}ke$ et. al. (1964), that the field would wrap up into a spiral similar to the spiral arms in only two or three galactic rotations. It was supposed that this is the natural shape of the magnetic field lines. If the winding up continued for the fifty or so rotations of the galaxy, the field lines would reverse in direction every one hundred parsecs. This seemed absurd. The various attempts to get around this problem involved magnetic fields which were strong enough to control the flow, and also strong radial outflows. Several forceful arguments against the primordial origin were advanced by Parker. The arguments were that the

field would be expelled from the galaxy, either by ambipolar diffusion(Parker 1968), or by rapid turbulent diffusion(Parker 1973a, 1973b)

In this paper we reexamine the problem of a a primordial field. We proceed along lines first initiated by Piddington. Just as in his model for the primordial field, we start with a cosmic field with lines of force threading the galaxy. We further assume that, after the collapse to the disk, the lines remain threaded through the galactic disk. The lines enter the lower boundary of the disk in a vertical direction, extend a short horizontal distance in the disk, and then leave through the upper boundary. Thus, each line initially extends a finite horizontal distance in the disk.

However, in contrast to Piddington's model, we assume the field is too weak to affect the plasma motions, especially the rotational motion about the galactic center. Consequently, it tends to be wrapped up by the differential rotation of the galaxy. In addition, we include additional physics of the interstellar medium in which the magnetic field evolves. After toroidal stretching strengthens the toroidal field sufficiently, a strong vertical force is exerted on the ionized part of the interstellar medium forcing it through the neutral part. We contend that this force will not expel the entire lines of force from the galactic disk, but the resulting ambipolar velocity will only tend to shorten the length which each line spends in the disk as it threads through it. In particular, ambipolar diffusion will decrease the radial component of the horizontal field and shorten its radial extent in the disk. This will decrease the toroidal stretching of the field line, and as a result the magnetic field strength will approach a slowly decreasing saturated condition. Thus, after several gigayears, the lines will end up extending a longer distance in the azimuthal direction than they did initially, but a much shorter distance in the radial direction. At any given time after the field saturates, the final strength will be independent of its initial value. In addition, the field strength will depend only on the ambipolar properties of the interstellar medium. [For earlier work see Howard (1995) and Kulsrud(1986, 1989, 1990).]

As a consequence of ambipolar diffusion plus stretching, the magnetic field will be almost entirely toroidal, and it will have the same strength everywhere at a fixed time. However, its sign will depend on the sign of the initial radial component of the field at its initial position. Because

of differential rotation the toroidal field will still vary rapidly in sign over a radial distance of about one hundred parsecs.

This model for the magnetic field evolution would seem at first to leave us with a wrap up problem and produce a field at variance with the observed field. However, the initial regions where the radial field is of one sign are expected to be of different area than those of the other sign provided that the initial field is not exactly uniform. As a consequence, at the present time in any given region of the galaxy, the toroidal field should have a larger extent of one sign than of the other, even, though the sign varies rapidly. If one now averages over larger regions than the size of variation, one would see a mean magnetic field of one sign. This averaging is actually performed by the finite resolution of the observations of the magnetic field in our galaxy or in external galaxies. From the observations one would not be aware of this rapid variation.

If one ignored the spiral arms, the field would be almost completely azimuthal. However, it is known that the density compression in the spiral density wave, twists the magnetic field to be parallel to the spiral arm and increases its field strength (Manchester 1974, Roberts and Yuan 1970). In observing radio emission from external galaxies, one tends to see the radiation mostly from the spiral arm, where cosmic rays are intense and the magnetic field is stronger. Because in these regions the field is aligned along the arms, one would naturally get the impression that the field extends along the entire arm. In fact, on the basis of our model, one would actually be seeing short pieces of field lines pieced together as they cross the spiral arm and thread through the disk. The magnetic field would be mainly azimuthal in between the spiral arms, and only as it crosses the arm would it be twisted to align along the arm. Observationally the magnetic field of this model would appear the same as that of a large scale magnetic field.

Similarly, one should see Faraday rotation which is proportional to the amount by which the effect of the toroidal field of one sign exceeds the other sign in this region. The amount of rotation produced by any one region would be related to difference in areas occupied by the fields of different sign. This in turn, is given by the amount by which the initial area at the initial position where the radial field is of one sign exceeds the initial area where the radial field is of

opposite sign.

In Parker's argument(Parker 1973a, 1973b) concerning the expulsion of the primordial field he introduces the concept of turbulent mixing. Turbulent mixing correctly describes the rate at which the mean field will decrease by mixing. However, it can not change the number of lines of force threading the disk, since this is fixed by their topology. It only gives the lines a displacement. Since, near the edge of the disk, the turbulent motions probably decrease as the sources of turbulence do, it is not expected that the lines will be mixed into the halo. Also, because of their topology the lines are not lost. Only the length of their extent in the disk can be altered by turbulence. Hence, for our model, turbulent diffusion need not destroy the primordial field as Parker suggested. Also, blowing bubbles in the lines by cosmic ray pressure will always leave the remainder of the line behind, and the total number of lines unaltered. Of course, if the lines were entirely horizontal, ambipolar diffusion could destroy the field, since the lines may be lifted out of the disk bodily(Parker 1968).

Therefore, Parker's contention that the primordial field has a short life, need not apply, to our model.

1.1. Our Model

The model which we consider is very simple. We start with a large ball of plasma whose mass is equal to the galactic mass, but whose radius is much larger than the current size of our galaxy (figure 1a). We first assume that the magnetic field filling this sphere is uniform, and makes a finite angle α with the rotation axis of the galaxy.

Then we allow the ball to collapse to a sphere the size of the galaxy (figure 1b), and finally to a disk the thickness of the galactic disk (figure 1c). We assume the first collapse is radial and uniform, while the second collapse into the disk is linear and one dimensional.

During the time when the galaxy contracts uniformly into the disk along the z direction, where $\hat{\mathbf{z}}$ is in the direction of $\mathbf{\Omega}$, we ignore any rotation. Then the resulting magnetic field

configuration is as in fig, (1c). The horizontal component of the magnetic field has been amplified by the large compressional factor, while the vertical component is unchanged, so that the resulting field is nearly parallel to the galactic disk. At this stage, some lines enter the disk from the top and leave from the top, e.g. line a. Some lines enter from the bottom and leave through the top, e.g. lines b and c. Finally, some lines enter from the bottom and leave through the bottom, e.g. line d. It turns out that lines such as a and d are eventually expelled from the disk by ambipolar diffusion, so that we ignore them. Now, set the disk into differential rotation at time $t = 0$. (If we include the rotation during collapse the magnetic field will start to wind up earlier. However, the result would be the same as though we were to ignore the rotation during collapse, and then after the collapse let the disk rotate by an additional amount equal to the amount by which the disk rotated during the collapse. This is true provided that the initial field is too weak for ambipolar diffusion to be important.)

1.2. Results of the Model

We here summarize the conclusions that we found from the detailed analysis of our model, given in the body of this paper.

Initially, after the disk forms, all the lines have a horizontal component that is larger than the vertical component by a factor equal to the radius of the disk divided by its thickness $\approx R/D \approx 100$. This is the case if the initial angle α was of order 45 degrees or at least not near 0 or 90 degrees.

Now, because of neglect of rotation in the collapse, the horizontal component of all the field lines is in a single direction, the x direction say, so that

$$\mathbf{B} = B_i\hat{\mathbf{x}} + B_i(D\tan\alpha/R)\hat{\mathbf{z}} = B_i\cos\theta\hat{\mathbf{r}} - B_i\sin\theta\hat{\theta} + B_i(D\tan\alpha/R)\hat{\mathbf{z}}. \quad (1)$$

After $t = 0$ the differential rotation of the disk stretches the radial magnetic field into the toroidal

direction so that, following a given fluid element whose initial angle is θ_1, one has

$$\mathbf{B} = -B_i \cos\theta_1 \hat{\mathbf{r}} + \left[B_i (r \frac{d\Omega}{dr} t) \cos\theta_1 - B_i \sin\theta_1 \right] \hat{\theta} + B_i (D \tan\alpha / R) \hat{\mathbf{z}}. \tag{2}$$

After a few rotations the second component dominates the first and third components. It is seen that the total magnetic field strength grows linearly with time. After the toroidal magnetic field becomes strong enough, the magnetic force on the ionized part of the disk forces it through the neutral component, primarily in the z direction.

Consider a single line of force. Let us turn off the differential rotation for a moment. In this case, the z motion steepens the line of force, but does not change the vertical field component. This leads to a shortening of the line, both in the radial direction and in the azimuthal direction (see figure 1d). Now, let the differential rotation continue. Because the radial component of the magnetic field is reduced, the toroidal component increases more slowly. Eventually, there comes a time when the radial field is small enough that the shortening motions in the azimuthal direction are stronger than the stretching motion, and the azimuthal component of the magnetic field actually decreases even in the presence of differential rotation. From this time on, the magnetic field strength decreases at a rate such that the vertical ambipolar velocity is just enough to move the plasma a distance approximately equal to the thickness of the galactic disk, in the time t, i. e.

$$v_D t = D. \tag{3}$$

Now, v_D is essentially proportional to the average of the square of the magnetic field strength. Therefore, for a uniform partially ionized plasma we have

$$v_D \approx \frac{1}{\rho_i \nu} \frac{B^2}{8\pi D}, \tag{4}$$

where D is the half thickness of the disk, ρ_i is the ion mass density, and where ν is the ion neutral collision rate.

Thus, for asymptotically long times one finds that

$$B \approx D\sqrt{\nu \rho_i / t}. \tag{5}$$

That is, the magnetic field strength approaches a saturated time behavior independent of its initial value. The saturated time behavior only depends on the ambipolar diffusion properties of the interstellar medium, and on the time t. However, the time to reach saturation does depend on the initial value of the magnetic field strength.

The qualitative behavior is shown in figure 2, where the dependence of B on time for different initial values is shown. For a very weak initial radial component of the field, saturation is not reached during a Hubble time. However, for fields substantially larger than the critical initial field strength for reaching saturation, the final saturated field is independent of the initial radial field.

In the interstellar medium, ambipolar diffusion is not well modeled by diffusion through a uniform plasma. In fact, the bulk of the mass of the interstellar medium is in dark clouds, in which the degree of ionization is very low. Also, the outward magnetic force is concentrated in the volume of the clouds. As a result the ambipolar diffusion velocity is more accurately given by the formula

$$v_D = \frac{B^2(1+\beta/\alpha)}{8\pi\rho_i\nu f D}, \tag{6}$$

where f is the filling factor for the clouds, ρ_i is the effective ion density in the clouds, ν is the effective ion-neutral collision rate in the clouds, and β/α is the ratio of cosmic ray pressure to magnetic pressure.

The model for the interstellar medium, which we employ to study the magnetic field evolution, is sketched in figure 3. The magnetic field is anchored in the clouds whose gravitational mass holds the magnetic field in the galactic disk. The magnetic field lines bow up in between the clouds pulled outward by magnetic pressure and by cosmic ray pressure. The outward force is balanced by ion–neutral friction in the clouds themselves. A derivation of equation 6 based on this model is given in section 5.

Taking plausible values for the properties of the clouds, one finds that the critical initial value of the magnetic field, for it to reach saturation in a Hubble time is about 10^{-8} gauss. Further, the saturated value of the field at $t = 10^{10}$ years is estimated to be 2 microgauss. At this field

strength the time to diffuse across the disk $D/v_d \approx 3 \times 10^9$ years, which is of order of the lifetime of the disk.

Finally, let us consider the structure of the saturated field. The time evolution of the magnetic field refers to the field in the rotating frame. Thus, if we wish the toroidal field at time t and at $\theta = 0$ we need to know the initial radial component of the field at $\theta_1 = -\Omega(r)t$. Thus, assuming that the field reaches a saturation value of B_S, one has

$$B_\theta = \pm B_S, \tag{7}$$

where the sign is plus or minus according to the sign of

$$[\cos(-\Omega(r)t)]. \tag{8}$$

Since Ω depends on r, we see that B_θ changes sign over a distance Δr such that $(\Delta\Omega)t = (\Delta r d\Omega/dr)t \approx \pi$, i.e.

$$\frac{\Delta r}{r} = \frac{-\pi}{r|d\Omega/dr|t} \approx \frac{\pi}{|\Omega|t}. \tag{9}$$

Taking the rotation period of the galaxy to be 200 million years, one finds for $t = 10^{10}$ years, $\Delta r \approx 100$ parsecs.

Therefore, because the field saturates to a constant field strength, it is predicted that a uniform initial field, that has a value greater than $> 10^{-8}$ gauss, immediately after the collapse to the disk, leads to a toroidal field that varies with r, at fixed θ, as a square wave with a reversal in sign every 100 parsecs or so. Such a field structure would produce no net Faraday rotation and would contradict observations.

However, this result can be traced to our assumption that the initial magnetic field in figure 1 was entirely uniform. Suppose it were not uniform. Then, after collapse, the radial field is not purely sinusoidal (see figure 4). In fact, one expects a behavior more like

$$B_r(t=0) = B_i[\cos\theta + \epsilon\cos 2\theta], \tag{10}$$

where ϵ is an index of the nonuniformity. As an example, for moderately small $\epsilon, (\cos\theta + \epsilon\cos 2\theta)$

is positive for

$$-\pi/2 + \epsilon < \theta < \pi/2 - \epsilon, \tag{11}$$

and negative for

$$\pi/2 - \epsilon < \theta < 3\pi/2 + \epsilon. \tag{12}$$

Therefore the radial extent in which $B_\theta(t)$ is negative is larger than the radial extent in which it is positive by a factor

$$\frac{\pi - 2\epsilon}{\pi + 2\epsilon} \tag{13}$$

The resulting field is positive over a smaller range of r than the range in which it is negative. Consequently, in such a magnetic field there would be a net Faraday rotation of polarized radio sources. Note that it is easily possible that the regions in which the radial field is initially weaker can end up as regions that dominate in flux over those region that come from regions in which the radial field was stronger! Now, if one averages this field over regions much larger than 100 parsecs, then the resulting mean field is smooth and axisymmetric. This result is contrary to the generally held belief that a primordial field should lead to a field with bisymmetric symmetry(Sofue et al 1986). This result alone shows that a more careful treatment of the evolution of the primordial field, such as that discussed above, leads to quite different results from those commonly assumed. (However, as mentioned above, including the effect of the spiral density wave will produce a magnetic field lines in the spiral arms that will appear to have bisymmetrical shape.)

We emphasize that, although the actual magnetic field derived in our model is a tightly wound spiral, the magnetic field averaged over a moderate size scale appears to be smooth and axisymmetric.

A second important conclusion of our model is that because of their topology, the field lines cannot not be expelled from the disk by ambipolar diffusion. The same line of force diffuses downward in the lower part of the disk, and upward in the upper part of the disk. The line must thus continue to be threaded through the disk. (See lines b and c in figure 1c.)

A third conclusion that can be drawn from our model is that any single line after saturation

has only a finite extent in the disk. For example, if the initial field is a few microgauss, then it turns out that the line only extends a radian or so before leaving the disk. This finite extent of the magnetic lines would make the escape of cosmic rays from the galaxy possible without the necessity of any disconnection of the magnetic field lines in the interstellar medium.

A final conclusion that should be noted is, that the saturated magnetic field of our model has a mean field strength smaller than the rms field strength. This has the consequence that different methods for measuring the magnitude of the magnetic field strength should lead to different results. The measurement by nonthermal radio emission of the cosmic ray electrons measures the rms field strength while Faraday rotation measures the mean strength.

The outline of the paper is as follows:

In section 2 an analytic model is developed to demonstrate the properties described in this introduction.

In section 3 a more precise one dimensional numerical simulation is carried out that confirms the evolution of the field in the z direction given in section 2.

In section 4 it is shown that the three dimensional equations for the evolution of the field can be reduced to two independent variables. These are z and an angular coordinate $u = \theta - \Omega(r)t$ that is constant along the spirals generated by the differential rotation of the galaxy. A numerical simulation of the resulting differential equations is carried out. It is shown that after a long time the resulting magnetic field does evolve locally in the essentially same way as is given in sections 2 and 3. In addition, it varies in radius as a square wave with uneven lobes.

In section 5 the astrophysics of the interstellar medium clouds is discussed, and a derivation is given of expression 6, for the effective mean ambipolar motion of the field in the disk.

In the concluding section 6, the implications of this model for the evolution of a magnetic field of primordial origin are given. The bearing of these implications on the various criticisms of the primordial field hypothesis are discussed.

2. Local Theory: Analytic

In this and the next section we wish to consider the local behavior of the magnetic field following a fluid element that moves with the galactic rotation. If the ambipolar diffusion were strictly in the z direction, the evolution of the field in a given fluid element would be independent of its behavior in other fluid elements at different r or θ. It would only be affected by the differential rotation of the galaxy, and by ambipolar velocity in the z direction. Thus, in particular, we could imagine the magnetic field at different values of θ, behaving in an identical manner. In other words, we could replace the general problem by an axisymmetric one.

Let

$$\mathbf{B} = B_r(r,z)\hat{\mathbf{r}} + B_\theta(r,z)\hat{\theta} + B_z(r,z)\hat{\mathbf{z}}. \tag{14}$$

Let us neglect the radial velocity, and let the ambipolar velocity be only in the $\hat{\mathbf{z}}$ direction and proportional to the z derivative of the magnetic field strength squared. Also, let us neglect any turbulent velocity. The only velocities which we consider are the differential rotation of the galaxy, $\Omega(r)$, and the ambipolar diffusion velocity of the ions.

Then

$$\mathbf{v} = r\Omega(r)\hat{\theta} + v_z\hat{\mathbf{z}}, \tag{15}$$

where

$$v_z = -K\frac{\partial B^2/8\pi}{\partial z}, \tag{16}$$

where

$$K = \frac{(1+\beta/\alpha)}{\rho_i f \nu}, \tag{17}$$

where as in the introduction β/α is the ratio of the cosmic ray pressure to the magnetic pressure, ρ_i is the ion density, and, ν is the ion-neutral collision rate in the clouds. The equation for the evolution of the magnetic field is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{18}$$

or, in components,

$$\frac{\partial B_r}{\partial t} = \frac{\partial}{\partial z}(v_z B_r), \tag{19}$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial z}(v_z B_\theta) + \frac{d\Omega}{dr} B_r, \tag{20}$$

$$\frac{\partial B_z}{\partial t} = \frac{\partial v_z}{\partial r} B_r. \tag{21}$$

We integrate these equations numerically in section 3. They are too difficult to handle analytically. To treat them approximately, we make the assumption that for all time B_r and B_θ vary parabolically in z i.e.

$$B_r = \quad B_r^0(1 - z^2/D^2), \tag{22}$$

$$B_\theta = \quad B_\theta^0(1 - z^2/D^2), \tag{23}$$

where B_r^0 and B_θ^0 are functions of time. We apply equation 19 and equation 20 at $z = 0$, and $r = r_0$, and solve for B_r^0, and B_θ^0 as functions of t.

Thus, making use of equations 16, 18 and 19, we find

$$\frac{\partial B_r}{\partial t} = -\frac{v_D}{D} B_r, \tag{24}$$

$$\frac{\partial B_\theta}{\partial t} = -\frac{v_D}{D} B_\theta + (r\frac{d\Omega}{dr})_{r_0} B_r, \tag{25}$$

$$v_D = \frac{K}{2\pi D}(B_r^2 + B_\theta^2), \tag{26}$$

where everything is evaluated at $z = 0, r = r_0$ so we drop the superscripts on B_r and B_θ. Since for galactic rotation $r\Omega$ is essentially constant, we have $(rd\Omega/dr)_{r_0} = -\Omega(r_0) \equiv -\Omega_0$.

Initially B_r and B_θ are of the same order of magnitude. Also, we expect B_θ to grow, by stretching, to a value much greater than its initial value, before ambipolar diffusion becomes important. Thus, for simplicity, we make the choice of initial conditions $B_r = B_1, B_\theta = 0$ where B_1 is the initial value for the radial component of the field. According to equation 2, $B_1 = B_i \cos\theta$ for a fluid element starting at θ, for a fluid element starting at θ.

Now, if initially $v_D/D \ll \Omega$, then according to equation 25, we expect B_θ to at first grow linearly in time, Then by equation 26, v_D increases quadratically in time till $v_D/D \approx \Omega$, and the ambipolar velocity starts to affect the evolution of B_θ and B_r. If the initial field is small, then the

contribution of B_r to the ambipolar diffusion velocity is never important. In this case we can write

$$v_D = v_{D1}\frac{B_\theta^2}{B_1^2}, \tag{27}$$

where $v_{D1} = KB_1^2/(2\pi D)$. The solution to the differential equation 24 and equation 25 with v_D given by equation 27 is

$$B_r = \frac{B_1}{(1+\frac{2v_{D1}}{3D}\Omega^2 t^3)^{1/2}}, \tag{28}$$

$$B_\theta = \frac{B_1 \Omega t}{(1+\frac{2v_{D1}}{3D}\Omega^2 t^3)^{1/2}}, \tag{29}$$

and

$$v_D = \frac{v_{D1}}{D}\frac{\Omega^2 t^2}{(1+\frac{2v_{D1}}{3D}\Omega^2 t^3)}. \tag{30}$$

(The above solution, equations 28 and 29, of equations 24, 25, and 27, is derived explicitly in Appendix A. However, it can be shown by direct substitution that it is a solution of equations 24, 25, and 27.)

From these equations we see that for

$$t \ll \left(\frac{3D}{2v_{D1}\Omega^2}\right)^{1/3}, \tag{31}$$

B_r is unchanged, B_θ increases as Ωt, and $\int v_D dt \ll D$, so that up to this time ambipolar diffusion carries the plasma only a small fraction of the disk thickness.

On the other hand, if

$$t \gg \left(\frac{3D}{2v_{D1}\Omega^2}\right)^{1/3}, \tag{32}$$

then

$$B_\theta \approx B_1\sqrt{\frac{3D}{2v_{D1}t}} = D\sqrt{\frac{6\pi}{Kt}} = D\sqrt{\frac{6\pi f \rho_i \nu}{(1+\beta/\alpha)t}}, \tag{33}$$

and B_θ is independent of both the initial value of B_r and of Ω. It depends only on the ambipolar diffusion properties of the interstellar medium.

For all t, we have

$$\frac{B_r}{B_\theta} = \frac{1}{\Omega t}, \tag{34}$$

so for $t = 10^{10}$ years, $B_r = B_\theta/300$, and the field becomes strongly toroidal.

The question arises as to how strong B_1 must be in order that the saturated solution, equation 33, is reached. Taking $t = t_H$, the Hubble time, and making use of the expression for v_{D1}, one finds from equation 32 that for saturation to be reached we must have

$$B_1 > \frac{D}{t_H^{3/2}} \sqrt{\frac{4\pi f \rho_i \nu}{3(1+\beta/\alpha)}}. \tag{35}$$

Making use of the numbers derived in section 5 one finds that if B_θ is saturated at $t = t_H$, then

$$B_\theta = \frac{D}{100\text{pc}} \left(\frac{10^{10}\text{years}}{t_h}\right)^{1/2} \left(\frac{6\times 10^{-4}}{n_i n_0}\right)^{1/2} 1.9\times 10^{-6}\text{gauss}, \tag{36}$$

where n_i is the ion density in the clouds, assumed to be ionized carbon, and n_0 is the mean hydrogen density in the interstellar medium. The densities are in cgs units.

For saturation, the critical value for B_1 is

$$B_{1crit} = \frac{D}{100\text{pc}} \left(\frac{10^{10}\text{years}}{t_h}\right)^{3/2} \left(\frac{6\times 10^{-4}}{n_i n_0}\right)^{1/2} 4\times 10^{-9}\text{gauss} \tag{37}$$

where the densities are in cgs units.

Hence, for the above properties of the clouds and for an initial radial field greater than the critical value, the magnetic field at $t = t_H$ saturates at about the presently observed value. Such a field arises from compression if, when the galaxy was a sphere of radius 10 kiloparsecs, the magnetic field strength was greater than 10^{-10} gauss. If before this the virialized radius of the protosphere was 100 kiloparsecs, then in this sphere the comoving initial value for the cosmic field strength had to be greater than 10^{-12} gauss. (That is if the cosmic field filled all space then it had to be so strong that the present value of the magnetic field in intergalactic space must now be 10^{-12} gauss.) There are good reasons to believe that a magnetic field stronger than this minimum value could have been generated by turbulence in the protogalaxy during its collapse to this virialized radius (Kulsrud et al. 1996). However, this field would be local to the galaxy and not fill all space.

Finally, one expects that $B_z \approx B_1/100$, from the initial compression into the disk. Taking B_z as unaffected by the differential stretching and by the ambipolar diffusion velocity, one can derive an expression for the length of a line of force:

$$\frac{rd\theta}{dz} = \frac{B_\theta}{B_z} = \frac{100\Omega t}{(1+\frac{2v_{D1}}{3D}\Omega^2 t^3)^{1/2}} \approx 100\sqrt{\frac{3D}{2v_{D1}t}}, \tag{38}$$

or

$$r\Delta\theta = \left(\frac{(2.5\times 10^{-6}\text{gauss}}{B_1}\right) 10\text{kpc}. \tag{39}$$

Thus, for example if $B_1 = 4.5\times 10^{-7}$ gauss, then a line of force passing through the solar position in the galaxy, would stretch once around the galaxy. Stronger initial fields lead to shorter lines of force. If the initial field is stronger than 2 microgauss, then it is possible that cosmic rays can escape along the lines of force into the halo during their average life time.

3. One Dimensional Numerical Simulation

It is clear that any gradient of the magnetic field strength will lead to ambipolar diffusion. However, gradients in the angular direction are weak, so that we may neglect ambipolar diffusion in the angular direction. Similarly, the gradients in the radial direction are weak at first, although as pointed out in the introduction, the magnetic field ends up reversing rapidly in the toroidal direction, so eventually radial ambipolar diffusion becomes as important as vertical ambipolar diffusion.

In this section, we restrict ourselves to gradients only in the $\hat{\mathbf{z}}$ direction. In this case, the axisymmetric approximation is valid, and the relevant equations are equation 19 and equation 20, where v_z is given by equation 16,

$$v_z = -K\frac{\partial}{\partial z}\frac{B^2}{8\pi}. \tag{40}$$

In the previous section, these equations were reduced to zero dimensions by the *ansatz* that B_r and B_z were parabolic in z according to equation 22 and equation 23, and the basic equations

were applied only at $z = 0$. In the present section we treat these equations numerically for all z with $|z| < D$, and drop the parabolic assumption. We treat the disk as uniform so that ρ_i and ν are taken as constants.

We need boundary conditions at $z = \pm D$. We expect B to be quite small outside the disk, $|z| > D$. (This is because we suppose that neutrals are absent in the halo and $\nu = 0$, so that the flow velocity v_z becomes very large. Since flux is conserved, vB must be a constant in a steady state and the magnetic field must be very small.) In order to match smoothly to the outer region, we first assume that ρ_i is a constant for all z and is very small. In addition, we assume that ν is constant for $|z| < D$, and decreases rapidly to zero in a narrow region, $D < |z| < D + \Delta$. Then because $\nu \approx 0$ in the halo region, $|z| > D$, v_z becomes so large that inertia is important.

The equation for v_z should read

$$\rho_i \left(\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial B^2/8\pi}{\partial z} - \rho_i \nu' v_z, \tag{41}$$

where we have set $\rho_i \nu' = (\rho_i \nu / f)(1 + \beta/\alpha)/f$. Because the time evolution is over the Hubble time t_H, $\partial v_z/\partial t \ll v_z \partial v_z/\partial z$, i.e. v_z/D is very large compared to $1/t_H$. Thus, we drop the partial time derivative.

The equations for B_r and B_θ are the same as in the disk. However, because v_z is large the divergence terms $\frac{\partial}{\partial z}(v_z B_r)$ and $\frac{\partial}{\partial z}(v_z B_\theta)$ are much larger than the other terms in the region near $|z| = D$, and in the halo. Thus, we have

$$\frac{\partial}{\partial z}(v_z B) = 0, \tag{42}$$

where $B = \sqrt{B_r^2 + B_\theta^2}$ is the magnitude of the magnetic field. This means that

$$vB = \Phi = \text{const}, \tag{43}$$

where Φ is the rate of flow of magnetic flux, and it is a constant in these regions. (We drop the subscript on v)

Dividing equation 41 by ρ_i and dropping the partial time derivative, we get

$$\frac{\partial}{\partial z} \left(v^2 + \frac{B^2}{4\pi\rho_i} \right) = -2\nu' v. \tag{44}$$

Combining this equation with equation 43 we have

$$\frac{\partial}{\partial z}\left(v^2+\frac{\Phi^2}{4\pi\rho_i\nu'^2}\right)=-2\nu' v, \tag{45}$$

or

$$\frac{d\left(v^2+\Phi^2/4\pi\rho_i v^2\right)}{v}, = -2\nu' dz, \tag{46}$$

or

$$-\int\left((\frac{\Phi^2}{4\pi\rho_i v^4}-1\right)dv=-\int \nu' dz. \tag{47}$$

For z larger than $D+\Delta$, the right hand side becomes a constant, so that $v^4 \to \Phi^2/4\pi\rho_i$. As z decreases below D, the right hand side becomes linear in z, and we find that for z far enough into the disk, the inertial term becomes negligible. Hence,

$$3v^3 \approx \frac{\Phi^2}{4\pi\rho_i\nu'(D-z)}, \tag{48}$$

and v approaches a small value. This result breaks down when v is small enough that the other terms in equation 19 and equation 20 become important.

Thus, the connection of the main part of the disk to the halo is given by equation 47

$$\int_v^{v_c}\left(\frac{\Phi^2}{4\pi\rho_i v^4}-1\right)dv=-\int_{z_c}^{z}\nu' dz, \tag{49}$$

where $v_c=(\Phi^2/4\pi\rho_i)^{1/4}$. Transforming this to an equation for B we have

$$\int_{B_c}^{B}\left(\frac{B^4}{4\pi\rho_i\Phi^2}-1\right)\frac{\Phi dB}{B^2}=-\int_{z_c}^{z}\nu' dz, \tag{50}$$

where $B_c=\Phi/v_c=(4\pi\rho_i\Phi^2)^{-1/4}$, and where z_c is the value of z where $B=B_c$. Let us write $\int_D^\infty \nu' dz=\nu_0'\Delta'$, where on the right hand side ν_0' denotes the constant value of ν' in the disk. Then for z far enough into the disk, equation 50 reduces to

$$\frac{B^3}{3(4\pi\rho_i)}\approx(D-\epsilon-z)\nu_0, \tag{51}$$

where

$$\epsilon=\Delta'+2v_c/3\nu_0. \tag{52}$$

Now, Δ' is small by assumption. If we estimate Φ as $B_0 v_{D\mathrm{eff}}$, where $v_{D\mathrm{eff}}$ is the order of the ambipolar diffusion velocity at the center of the the disk, then $v_c \approx (B_0 v_{D\mathrm{eff}}/\sqrt{4\pi\rho_i})^{1/2} \ll B_0/\sqrt{4\pi\rho_i}$, so that ϵ is much smaller than the distance an alfven wave can propagate in the ion-neutral collision time. This is clearly a microscopic distance compared to the thickness of the disk, so that we may neglect it.

In summary, equation 50 implies that the inner solution essentially vanishes at $z \approx \pm D$, so that we may take our boundary condition to be $B_r = B_\theta = 0$ at $z = D$. Given this solution the above analysis shows how to smoothly continue it into the halo.

Equation 19 and equation 20 can be made dimensionless by a proper transformation. We choose a transformation that is consistent with that employed in the next section. Define a unit of time t_0 by

$$\Omega_0 t_0 = R/D, \tag{53}$$

where R is the radius of the sun's galactic orbit, and $\Omega_0 = \Omega(R)$ is its angular velocity. Next, choose a unit of magnetic field B_0 to satisfy

$$\frac{KB_0^2}{4\pi D} = \frac{D}{t_0}. \tag{54}$$

For such a field the ambipolar diffusion velocity is such that the field lines cross the disk in a time t_0. Now, let

$$\begin{aligned} t &= t_0 t', \qquad (55) \\ B_r &= (D/R) B_0 B_r', \\ B_\theta &= B_0 B_\theta', \\ z &= D z'. \end{aligned}$$

For a cloud ion density of $6 \times 10^{-4}/\mathrm{cm}^3$, and a mean interstellar medium density of $n_0 = 10/\mathrm{cm}^3, B_0 = 2.85 \times 10^{-6}$gauss, (see section 5). Also, $D = 100$ parsecs, and $t_0 = 3 \times 10^9$ years. The Hubble time in these dimensionless units is about 3.

The details of the numerical simulation are given in another paper, (Howard, 1996). The dimensionless equations to be solved are

$$\frac{\partial B_r'}{\partial t'} = \frac{1}{2}\frac{\partial}{\partial z'}(\frac{\partial B'^2}{\partial z'}B_r'), \tag{56}$$

$$\frac{\partial B_\theta'}{\partial t'} = \frac{1}{2}\frac{\partial}{\partial z'}(\frac{\partial B'^2}{\partial z'}B_\theta') - B_r'. \tag{57}$$

Our dimensionless boundary condition is $B_r' = B_\theta' = 0$ at $z = \pm 1$.

The results of the integration of these equations are shown in figures 5 and 6. The initial profiles of B_r and B_θ are parabolic.

In figure 5 the initial value of B_r' is $B_i = 0.01$, so that $B_r = 2.85 \times 10^{-8}$ gauss. It is seen that the qualitative behavior is the same as that described in sections 1 and 2. The field at first grows and then relaxes back to a decaying solution. Figure 6 gives the time evolution of B_θ' at $z = 0$ for the set of initial conditions, $B_r = B_i \cos theta$ where θ runs from zero to one hundred eight degrees in increments of fifteen degrees. It is seen that the curves all tend to the same asymptotic behavior and are similar to those in figure 2. As in figure 2, the field from weaker initial values of B_r takes longer times to reach its peak value and the saturation curve. These results are similar to those which are represented by equations 28 and 29.

It is seen in figure 5 that, except near the edge, the profile remains similar to a parabola. Near the edge the cube root behavior of equation 51 is also evident. This cube root behavior can be understood since the flux $v_z B$ is roughly constant, and therefore $(\partial B'^2/\partial z')B' \approx (\partial/\partial z')(1-z')^{2/3} \times (1-z')^{1/3} \approx (1-z')^0 =$ a constant.

The most important feature in figure 5 and 6, is the saturation of B to the envelope curve. This saturation occurs if the initial value of B_r' is larger than 0.01. Thus, even in our more precise calculations, information on the initial B is lost, and the final value of B at fixed time depends only on the initial sign of B_r.

4. Two Dimensional Analysis

In the last section we have treated the evolution of the magnetic field under the influence of differential rotation and ambipolar diffusion in a one dimensional approximation. Only the z component of the ambipolar diffusion was kept. In this approximation the magnetic field in each column of fluid at r, θ evolves independently of any other r, θ column of fluid.

In early times, because the galactic disk is thin compared to its radius, this is a good approximation, since the horizontal gradients of B^2 are small compared to the vertical gradients. However, because of the differential rotation of the galaxy, fluid elements at different radii, that were initially very far from each other are brought much closer together, and the horizontal gradients are increased until they become comparable to the vertical gradients.

For example, two fluid elements that were initially on opposite sides of the galaxy, and at a difference in radius of one hundred parsecs, will be brought to a position on the same radius after about fifty rotations. Since the evolution of the field in these two fluid elements is very different, we expect the gradient of B^2 in the radial direction to become as large as that in the vertical direction. As a consequence, ambipolar drift velocities in the horizontal direction can be expected to be as large as those in the vertical direction. On the other hand, two fluid elements at the same radius, but initially far apart, remain far apart. The ambipolar motions in the θ direction should remain small.

Thus, we expect that, at first, only ambipolar z motions are important for the evolution of the magnetic field, but that eventually the radial ambipolar motions also will become important, although not the angular ambipolar motions. That is, we expect the problem to become two dimensional.

In order to properly demonstrate this evolution, we introduce a new independent variable

$$u \equiv \theta - \Omega(r)t. \tag{58}$$

If ambipolar diffusion is neglected, this variable is just the initial angular position of a fluid element that is at the position r, θ, z at time t. Since in the absence of ambipolar diffusion the

variables r, u, and z are constant, following a given fluid element. They would be the Lagrangian variables if only rotational motion is considered. It is appropriate to describe the evolution of the magnetic field components B_r, B_θ, and B_z in terms of these variables.

Inspection of equation 58 shows that when Ωt is large, u varies rapidly with r at fixed θ, in agreement with the above qualitative discussion. Changing r by only a small amount will change the initial angular position by π. Thus, we expect that the components of $\mathbf{B}$ will vary finitely with u, but only slowly with r for fixed u. The surfaces of constant u are tightly wrapped spirals. Thus, the behavior of the field should be finite in r, u, and z, but only gradients with respect to u and z should be important.

To see this, let us first write the total velocity, $\mathbf{v} = \mathbf{w} + \Omega r \hat{\theta}$, where $\mathbf{w}$ is the ambipolar velocity. Next, let us derive the equations for the components B_r, B_θ, B_z and w_r, w_θ, w_z in terms of the Eulerian variables r, θ and z.

$$\frac{\partial B_r}{\partial t} = (\mathbf{B} \cdot \nabla) w_r - (\mathbf{w} \cdot \nabla) B_r - B_r (\nabla \cdot \mathbf{w}) - \Omega \frac{\partial B_r}{\partial \theta}, \tag{59}$$

$$\frac{\partial B_\theta}{\partial t} = (\mathbf{B} \cdot \nabla) w_\theta - \frac{B_\theta w_r}{r} (\mathbf{w} \cdot \nabla) B_\theta + \frac{B_r w_\theta}{r} - \Omega \frac{\partial B_r}{\partial \theta} - B_\theta (\nabla \cdot \mathbf{w}), \tag{60}$$

$$\frac{\partial B_z}{\partial t} = (\mathbf{B} \cdot \nabla) w_z - (\mathbf{w} \cdot \nabla) B_z - \Omega \frac{\partial B_z}{\partial \theta} - B_z (\nabla \cdot \mathbf{w}), \tag{61}$$

where the ambipolar velocities are:

$$w_r = -K \frac{\partial B^2/8\pi}{\partial r}, w_\theta = -K \frac{\partial B^2/8\pi}{r \partial \theta}, w_z = -K \frac{\partial B^2/8\pi}{\partial z}, \tag{62}$$

and

$$\nabla \cdot \mathbf{w} = \frac{\partial w_r}{\partial r} + \frac{\partial w_\theta}{r \partial \theta} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z}. \tag{63}$$

Finally, let us transform these equations to the new coordinates r, u, z. In doing this we assume that the galactic rotation velocity $v_c = \Omega r$ is a constant.

The result of the transformation is

$$\begin{aligned} \frac{\partial B_r}{\partial t} &= \frac{B_\theta}{r} \frac{\partial w_r}{\partial u} + B_z \frac{\partial w_r}{\partial z} \\ &\quad - w_r \frac{\partial B_r}{\partial r} - \frac{w_\theta}{r} \frac{\partial B_r}{\partial u} - \Omega t \frac{w_r}{r} \frac{\partial B_r}{\partial u} - w_z \frac{\partial B_r}{\partial z} \end{aligned} \tag{64}$$

$$-B_r\left(\frac{w_r}{r}+\frac{\partial w_\theta}{r\partial u}+\frac{\partial w_z}{\partial z}\right),$$

$$\begin{aligned}\frac{\partial B_\theta}{\partial t} &= B_r\frac{\partial w_\theta}{\partial r}+\frac{\Omega t}{r}B_r\frac{\partial w_\theta}{\partial u}\\ &+B_z\frac{\partial w_\theta}{\partial z}-w_r\frac{\partial B_\theta}{\partial r}-\frac{w_\theta}{r}\frac{\partial B_\theta}{\partial u}-\frac{\Omega t}{r}w_r\frac{\partial B_\theta}{\partial u}\\ &-w_z\frac{\partial B_\theta}{\partial z}-\Omega B_r\\ &-B_\theta\frac{\partial w_r}{\partial r}-\frac{\Omega t}{r}B_\theta\frac{\partial w_r}{\partial u}-B_\theta\frac{\partial w_z}{\partial z},\end{aligned} \tag{65}$$

$$\begin{aligned}\frac{\partial B_z}{\partial t} &= B_r\frac{\partial w_z}{\partial r}+\frac{B_\theta}{r}\frac{\partial w_z}{\partial u}+\Omega t\frac{B_r}{r}\frac{\partial w_z}{\partial u}\\ &-w_r\frac{\partial B_z}{\partial r}-\frac{w_\theta}{r}\frac{\partial B_z}{\partial u}-\Omega t\frac{w_r}{r}\frac{\partial B_z}{\partial u}\\ &-w_z\frac{\partial B_z}{\partial z}-B_z\frac{\partial w_r}{\partial r}\\ &-\Omega t\frac{B_z}{r}\frac{\partial w_r}{\partial u}-B_z\frac{w_r}{r}-\frac{B_z}{r}\frac{\partial w_\theta}{\partial u},\end{aligned} \tag{66}$$

$$w_r=-K\frac{\partial B^2/8\pi}{\partial r}-K\frac{\Omega t}{r}\frac{\partial B^2/8\pi}{\partial u}, \tag{67}$$

$$w_\theta=-\frac{K}{r}\frac{\partial B^2/8\pi}{\partial u}, \tag{68}$$

$$w_z=-K\frac{\partial B^2/8\pi}{\partial z}. \tag{69}$$

Only a few of these terms are important. To see this let us introduce dimensionless variables for the velocity and field components. (We will denote the dimensionless variables by primes.) We will choose these variables based on our one-dimensional results, as follows:

The unit of length for the z variable is the galactic disk thickness D.

$$z=Dz'. \tag{70}$$

The variation of quantities with r is finite over the distance R the radius of the sun's orbit in the galaxy, so we set

$$r=Rr'. \tag{71}$$

The variable u is already dimensionless and quantities vary finitely with it, so we leave it unchanged.

The unit of time t_0 should be of order of the age of the disk. During this time the number of radians through which the galaxy rotates, Ωt is of the same order of magnitude as the ratio R/D, so for analytic convenience we choose t_0 so that

$$\Omega_0 t_0 = R/D, \tag{72}$$

and set

$$t = t_0 t'. \tag{73}$$

If we take $R/D = 100$, and $\Omega_0 = 2\pi/(2 \times 10^8 \text{years})$, then $t_0 = 3 \times 10^9$ years.

It is natural to choose the unit for B_θ, B_0 as that field whose z gradient produces an ambipolar z velocity of order D/t_0, that is an average velocity near that which would be produced by the saturated field. Thus, we choose B_0 so that

$$KB_0^2/(4\pi D) = D/t_0, \tag{74}$$

and set

$$B_\theta = B_0 B'_\theta. \tag{75}$$

(Note that the definitions in these units are consistent with those of section 3.)

In most cases of interest to us, B_r and B_z are much smaller than B_0. (In general B_θ is initially small compared to B_0, but it grows by a factor of R/D to up to the saturated value of about B_0.) Thus, we set

$$B_r = (D/R) B_0 B'_r. \tag{76}$$

The vertical field B_z starts out even weaker than this magnitude since the initial horizontal components of the magnetic field were amplified by the initial compression which formed the disk. However, the B_z field is amplified up to the size of the B_r field by the shear of the radial ambipolar velocity acting on B_r . We thus transform B_z by

$$B_z = (D/R) B_0 B'_z. \tag{77}$$

The ambipolar velocities w_r, w_θ, and w_z arise from gradients of the magnetic field in the corresponding directions. Thus, after B_θ has been amplified by stretching to be of order $B_0, w_z \approx KB_0^2/D$. Similarly, after the differential rotation has acted to reduce the scale of the variation of B^2 in the radial direction, w_r becomes of order w_z. However, the scale of variation in the θ direction remains of order R over the age of the galaxy so that $w_\theta \approx KB_0^2/R$. Thus, we change the w components to w' components by

$$\begin{aligned} w_r &= (D/t_0)w'_r, \\ w_\theta &= (D^2/Rt_0)w'_\theta, \\ w_z &= (D/t_0)w'_z. \end{aligned} \tag{78}$$

Now if we transform equations 64 to 69 by the change of variables equations 70 to 78, and clear the dimensional factors t_0, B_0 etc. from the left hand side, we find that the terms on the right hand side are either independent of dimensional units entirely, or are proportional to powers of $(D/R) << 1$. The full equations are given in appendix B. Dropping these "smaller" terms proportional to a power of D/R greater than zero, we find that the equations for the dimensionless variables, to lowest order in D/R are

$$\begin{aligned} \frac{\partial B'_r}{\partial t'} &= -\frac{1}{r'^2}w'_r t'\frac{\partial B'_r}{\partial u} - \frac{\partial(w'_z B'_r)}{\partial z'} \\ &\quad +\frac{B'_\theta}{r'}\frac{\partial w'_r}{\partial u} + B'_z\frac{\partial w'_r}{\partial z'}, \end{aligned} \tag{79}$$

$$\begin{aligned} \frac{\partial B'_\theta}{\partial t'} &= -\frac{t'}{r'^2}\frac{\partial(w'_r B'_\theta)}{\partial u} - \frac{\partial w'_z B'_\theta}{\partial z'} \\ &\quad -\frac{B'_r}{r'}, \end{aligned} \tag{80}$$

$$\begin{aligned} \frac{\partial B'_z}{\partial t'} &= \frac{B'_\theta}{r'}\frac{\partial w'_z}{\partial u} + \frac{t' B'_r}{r'^2}\frac{\partial w'_z}{\partial u} \\ &\quad -\frac{t' w'_r}{r'^2}\frac{\partial B'_z}{\partial u} - w'_z\frac{\partial B'_z}{\partial z} - \frac{t'}{r'^2}B'_z\frac{\partial w'_r}{\partial u}, \end{aligned} \tag{81}$$

$$w'_r = -\frac{t'}{2r'^2}\frac{\partial B'^2_\theta}{\partial u}, \tag{82}$$

$$w'_\theta = -\frac{1}{2}\frac{\partial B'^2_\theta}{\partial u}, \tag{83}$$

$$w'_z = -\frac{1}{2}\frac{\partial B'^2_\theta}{\partial z'}. \tag{84}$$

Note that w'_θ does not occur in the equations for the evolution of the B' components. Also, the r' derivatives are absent from these lower order equations. The initial conditions on $B'(t', u, z'; r')$ are

$$\begin{aligned} B'_r(0, u, z', r') &= B_r(0, r'R, u, z'D), \\ B'_\theta(0, u, z', r') &= B_\theta(0, r'R, u, z'D), \\ B'_z(0, u, z', r') &= B_z(0, r'R, u, z'D). \end{aligned} \tag{85}$$

These transformations are formal, but they enable us to correctly drop the terms whose effect is small. Once these terms are dropped, the equations reduce to two dimensional equations, which are more easily handled numerically. Although we have assumed that the dimensionless variables are originally of order unity, it may be the case that they differ substantially from unity. However, an examination of the various possible relevant cases leads to the conviction that all the important terms have been kept as well as other terms which are, perhaps, unimportant. For example, the initial value of B'_θ is much smaller than unity. However, because of the shearing terms in equation 80 (the last term) B'_θ grows to finite order when t' becomes of order unity, so during the later stages of the galactic disk, B'_θ is of order unity.

Many of the terms in equations 79 to 81 have an obvious significance. The second term on the right hand side of the B'_θ equation is the vertical decompression term present in the one dimensional simulation. The first term is the radial decompression term, which only becomes important when $t' \approx 1$, and the wrapping up has made the radial ambipolar diffusion important.

Similarly, there is a z decompressional term in the B'_r equation, but, of course, no radial decompression term. There is a term representing the effect of shear on the toroidal field in increasing the radial component, and a similar term resulting from the action of shear on the B'_z component. These shear terms would be small if B_θ were of order of B_r, or if B_z were much

smaller than B_r, which is the case initially. However, B_z is increased by shear terms over the age of the disk, to a value considerably larger than its initial value.

Equations 79 to 84 only contain derivatives with respect to t', z' and u, and none with respect to r'. Thus, r' is only a parameter in these differential equations. Therefore, the components of the magnetic field evolve independently of those at a different value of r' (to lowest order in D/R).

Further, u does not occur explicitly in the differential equations. The initial conditions, equations 85 do involve u, and are periodic in it. Therefore, the magnetic field components remain periodic in u for all t'.

Let us consider the behavior of such a solution, periodic in u, in the neighborhood of the sun, $r' = 1$, at fixed z', say $z' = 0$. Transform the solution back to r, θ coordinates. For fixed r and t, the solution is periodic in θ, e.g. $B_\theta(r, \theta) = B_\theta(r, u + \Omega(r)t)$. Moreover, for fixed θ we can write

$$\begin{aligned} u &= \theta - \Omega(r)t = \theta - \frac{\Delta r}{r}\Omega_0 t \\ &= \theta - \frac{\Delta r}{r} t' \frac{R}{D} = \theta - \frac{\Delta r t'}{D}, \end{aligned} \tag{86}$$

so

$$\Delta r = \theta - u\frac{D}{t'}. \tag{87}$$

Thus, for fixed θ, r changes by an amount $2\pi D/t'$ when u changes by its periodic length 2π. Since r' changes by a small amount $\approx 2\pi D/Rt'$, we may ignore the dependence of the solution for the components of $\mathbf{B}$ on r' and the components of $\mathbf{B}$ are nearly periodic in r (at fixed θ). However, because of the actual dependence of the solution on r', as a parameter in the equations, the amplitude and phase (as well as the shape) of the periodic solution do change slowly when one goes a distance comparable with the radius of the galaxy.

Equations 79 to 84 were integrated numerically. The details of the integration are discussed in Howard(1995, 1996), where most of the results are presented. Initial conditions were set by starting with a cosmic field before compression into the disk, and then calculating the resultant fields. In this paper we present the results for two initial cases. Only the results for the integrations

at $r = R$, the radius of the galactic solar orbit, are included. The variation of B_θ as a function of u, at $z = 0$, is plotted in figure 7 for the case that the initial cosmic field was uniform. The antisymmetry in u is evident, and it is clear, after transforming the field to be a function of r as the independent variable by equation 87, that no Faraday rotation would be produced by this field. The same result for the case when the initial cosmic field was nonuniform is shown in figure 8. [The initial cosmic field was chosen so after compression into the disk the horizontal field was $\mathbf{B} = B_i[.5 + x)\hat{\mathbf{x}} + y\hat{\mathbf{y}})$.] The resulting saturated field is not antisymmetric, and does not average out in u. It also would not average out when transformed to be a function of r, and *would* produce a Faraday rotation. The variation of B_θ with z at $u = 0$ at a time of 9 gigayears, is shown in figure 9. It has the parabolic shape found in section 3. The variation of B_θ with time at the point $u = 0, z = 0$, is given in figure 10 for the two cases. The results are also similar to those of figure 2 derived from the simple parabolic approximation.

5. Ambipolar Diffusion in the Interstellar Medium

We now consider the averaged equations for the magnetic field, taking into account the interstellar clouds. The bulk of interstellar matter is in the form of diffuse clouds and molecular clouds. Because the properties of the molecular clouds are not very well known, we make the simplifying assumption that essentially all the interstellar matter is in diffuse clouds, with a small amount of matter in the intercloud region. In describing the clouds, we make use of properties given by Spitzer(1968).

We assume that all the clouds are identical. We further include the cosmic ray pressure, and the magnetic pressure, in the intercloud region, but neglect the pressure of the intercloud matter. Then the cosmic rays and the magnetic fields are held in the disk against their outward pressures by the weight of the clouds in the gravitational field of the stars. (See figure 3.)

Now the force due to the magnetic and cosmic ray pressure gradients is exerted only on the ionized matter in the clouds, while the gravitational force is exerted mainly on the neutrals in the

clouds, since the fraction of ionization in the clouds is generally very low. Thus, these contrary forces pull the ions through the neutrals with ambipolar diffusion velocity v_D. The frictional force between the ions and neutrals is proportional to v_D. By equating the magnetic plus the cosmic ray force to the frictional force, we can obtain the mean ambipolar velocity in the clouds. Now, we assume that the cosmic ray pressure p_R is related to the magnetic pressure $B^2/8\pi$ by the factor β/α(Spitzer 1968). We take β/α independent of time and space. This is plausible since when the magnetic field is strong, we expect the cosmic ray confinement to be better and therefore the cosmic ray pressure to be larger.

The mean vertical force per unit volume produced by the magnetic field strength gradients and cosmic ray pressure gradients is

$$F = -(1 + \beta/\alpha)\frac{\partial B^2/8\pi}{\partial z}. \tag{88}$$

But this force is counterbalanced by the gravitational force on neutrals in the clouds, which occupy a fractional volume equal to the filling factor, f, times the total volume.

Thus, the force per unit volume on the ions in the clouds is

$$F_{cloud} = \frac{F}{f} = -(1 + \beta/\alpha)\frac{1}{f}\frac{\partial B^2/8\pi}{\partial z}. \tag{89}$$

This force produces an ambipolar velocity, v_D, of the ions relative to the neutrals such that

$$F_{cloud} = n_i m^* \nu v_D, \tag{90}$$

where m^* is the mean ion mass, ν is the effective ion–neutral collision rate for momentum transfer and n_i is the ion number density, in the clouds. Now,

$$\nu \approx n_c \frac{m_H}{m^* + m_H} < \sigma v > \tag{91}$$

where σ is the momentum transfer collision cross section, and we assume that the neutrals are all hydrogen with atomic mass m_H. Thus, we have

$$F_{cloud} = n_i \frac{m_H m^*}{m^* + m_H} < \sigma v > n_c v_D \tag{92}$$

If $m^* >> m_H$, then the mass factor is m_H. Hence, if the ions are mostly singly charged carbon, the mass factor is $\approx m_H$, while if they are mostly hydrogen ions then it is $m_H/2$. The ions are tied to the magnetic field lines, since the plasma is effectively infinitely conducting. If there are several species of ions, they all have the same cross field ambipolar velocity.

Thus, the ambipolar diffusion velocity in the clouds is

$$v_D = -\frac{1}{f}\frac{(1+\beta/\alpha)\nabla(B^2/8\pi)}{n_i m^*_{eff} < \sigma v > n_c}, \tag{93}$$

where $m^*_{eff} =< m^* m_H/(m^* + m_H) >$ is the effective mass averaged over ion species, and σ is the ion–neutral cross section.

The velocity v_D is the mean velocity of an ion inside a cloud. It is also the velocity of a given line of force. Now, the ions are continually recombining and being replaced by other ions, so it is actually the motion of the magnetic lines of force that has significance. Further, the clouds themselves have a short life compared to the age of the disk. They collide with other clouds every 10^7 years or so, and then quickly reform. After the collision the cloud material is dispersed, but because of its high conductivity, it stays connected to the same lines of force. After the cloud reforms the lines of force are still connected to the same mass. The lines then continue to move in the reformed cloud, again in the opposite direction to the field gradient. As a consequence, during each cloud lifetime, the lines of force pass through a certain amount of mass before the cloud is destroyed.

The amount of mass passed during a cloud life time, by a single flux tube of diameter d is $2v_D t_c a d \rho_c$ where t_c is the cloud life time, $2a$ is the cloud diameter, and ρ_c is the cloud density. However, because of our assumption that the bulk of the matter is in these diffuse clouds, it must be the case that there is only a short time in between the destruction of one cloud, and its reformation. Thus, during a time t, the field passes through $\approx t/t_c$ successive clouds. If we consider a length of a line of force L, at any one time it passes through $fL/2a$ different clouds. Then, in the time t, the amount of matter passed by this length of a given tube of force (of

diameter d) is

$$\Delta M = 2adv_D t_c \rho_c \frac{L}{2a}\frac{t}{t_c} = Lf\rho_c dv_D t. \tag{94}$$

But $f\rho_c$, is the mean density, ρ, of interstellar matter. Thus, if we define Δx as the average effective distance that this magnetic tube moves through the disk by

$$\Delta M = \rho \Delta x L d, \tag{95}$$

we get

$$d\rho\Delta x L = Ld\rho v_D t, \tag{96}$$

or

$$\frac{\Delta x}{t} = v_D. \tag{97}$$

This effective *velocity*, averaged over many cloud lifetimes, is the only velocity for the magnetic field lines that makes sense. It is equal to the velocity, v_D, of the ions in the cloud material.

So far, we have made the simplifying assumption that the clouds are stationary, which is of course not the case. As the clouds move through the interstellar medium they stretch the lines, and additional tension forces and ambipolar velocities arise. However, these velocities are always directed toward the mean position of the cloud material, and thus they tend to average to zero. The ambipolar velocity which we have calculated above, under the assumption of stationary clouds, actually gives the rate of displacement of the lines of force relative to the mean position of the cloud. It is a secular velocity, and it is the only velocity which really counts.

We assume that in the diffuse interstellar clouds only carbon is ionized, so that $m*_{eff} = m_H$. We choose $< \sigma v >= 2 \times 10^{-9}\mathrm{cm}^2/\mathrm{sec}$(Spitzer 1978). If the filling factor of the clouds $f = 0.1$, and if the mean interstellar density of hydrogen is $n_0 \approx 1.0/\mathrm{cm}^3$, then the density in the clouds is $n_c \approx 10/\mathrm{cm}^3$. The cosmic abundance of carbon is 3×10^{-4}(Allen 1963). We assume that this abundance is depleted by $\zeta_0 \approx 0.2$, (Spitzer 1978), the rest of the carbon being locked up in grains. We further take $\beta/\alpha = 2$. Then comparing equation 40 with equation 93 gives

$$K = \frac{(1 + \beta/\alpha)}{n_i m*_{eff} < \sigma v > n_0} = 1.5 \times 10^{36}, \tag{98}$$

in cgs units. Taking $D = 100$ pc, we get from equation 54

$$B_0 = 2.85 \times 10^{-6} \text{gauss}. \tag{99}$$

We take $t_0 = 3 \times 10^9$ years. We see that if $t' = 3$, corresponding to an age for the galactic disk of 9×10^9 years, then the present value of the magnetic field from figure 8, should be 1.5×10^{-6} gauss.

This value depends on our assumptions concerning the properties of the clouds. In particular, if the hydrogen in the interstellar clouds is partly ionized by low energy cosmic rays penetrating the clouds, then the ambipolar diffusion will be slower, and K will be smaller, leading to a larger value for B_0. The consequence of this is, that the initially dimensionless magnetic field B_r' will be smaller, so that it would take longer to reach saturation. However, the saturated value will be larger.

6. Conclusion

We have assumed that there was a cosmic magnetic field present before the galaxy formed. On the basis of this hypothesis we have constructed a simplified model of the galactic disk in order to investigate how the magnetic field would evolve, and what it would look like at present. The two essential ingredients of this model are: the differential rotation of the interstellar medium, and the motion of the field lines through the interstellar medium produced by the ambipolar diffusion of the ionized component of the plasma driven through the neutrals by magnetic pressure and cosmic ray pressure gradients. The effect of turbulent motions is assumed to average out. No large scale mean field dynamo action on the magnetic field was included in this model. (This model differs from that of Piddington in that the magnetic field is too weak to effect the galactic rotation of the interstellar medium, and also, ambipolar diffusion is included.)

The consequences of the model were investigated by an approximate analytical calculation in Section 2, and by more detailed numerical simulations in Section 3 and 4. These simulations

confirmed the results of the approximate analysis of section 2.

The basic results are:

(1) To first approximation, the magnetic field evolves locally following a rotating fluid element. It first grows by stretching the radial component of the magnetic field into the toroidal direction. When the field becomes strong enough, the line commences to shorten because of the vertical motions produced by ambipolar diffusion. This reduces the radial component, and therefore the stretching. After a certain time, the field strength saturates and starts to decrease as the reciprocal square root of time. This asymptotic behavior is determined only by the ambipolar diffusion properties of the clouds. Thus, the field strength everywhere approaches the same value at a given time. At the present time, this value is estimated to be in the range of a few microgauss. This saturated value is independent of the initial value which the field had when the disk first formed, provided that the initial value of the field strength is greater than 10^{-8} gauss. The extent of each magnetic field line in the toroidal direction also saturates, but its length in the disk *does* depend on the initial value.

(2) The direction of the toroidal field in any given fluid element depends on the sign of the initial radial component. Since this sign varies with position, and since differential rotation mixes these positions, it turns out that the resulting toroidal field varies rapidly with radius along a fixed radial direction. The toroidal field changes direction on a scale of a hundred parsecs. Because the saturated field strength is nearly constant in magnitude, the toroidal field strength as a function of r at fixed θ varies as a square wave. However, the lobes of this square wave need not be equal since the regions of one initial sign of B_r may be larger than those of the other sign. In this case, the model predicts a toroidal field that would produce a net Faraday rotation in radio sources such as pulsars or polarized extragalactic radio sources, in spite of its rapid variation in sign.

It is the prevailing belief that the galactic magnetic field does not reverse in radius on small scales. In fact, this belief is grounded in an analysis of Faraday rotation measures of pulsars(Hamilton and Lynn 1987). In analyzing these rotation measure Rand and Kulkarni(1988) employed various simple models of the galactic field. In every one of these models, the magnetic

field reversed only on large scales of order one kiloparsec. These models, which only allowed a slow variation of B, led to results that were consistent with the observations, and thus supported the general belief in reversal only on large scales. However, when this analysis was carried out, there was no apparent reason reason not to consider a model in which the field varied rapidly in radius on scales of order of a hundred parsecs, although in hindsight it could have been done. However, such a model was not included in the analysis of the rotation measures, and thus it was not tested. Therefore, at present, there is no reason to exclude such a model.

In short, the prevailing belief that the galactic magnetic field is of constant sign on a large scale, actually resulted from the assumption that the field was a large scale field, and from the consistency of this assumed model with observations. Therefore, this conclusion has not been rigorously demonstrated. A magnetic field, such as that arrived at from our model, which has the rapid variation of the field with radius, would lead to fluctuations from the mean. Indeed, such fluctuations are in the data and are attributed to a general isotropic random magnetic field in addition to the mean field.

We feel the the non uniform square wave could probably fit the observations equally as well as the other models, so that our model can also be shown to be consistent with observations. We have not yet demonstrated quantitative consistency with observations, but we hope to carry out this task in the future.

(3) The magnetic field observed in our galaxy and in other galaxies should actually be the average of the true detailed magnetic field averaged over regions in space larger than those regions over which we find our field to vary (several hundred parsecs at least). Thus, under averaging the field of our model would actually appear as an axisymmetric toroidal magnetic field. An analysis of the various galactic magnetic fields has made the *ansatz* that the origin of the field can be distinguished as primordial if it has bisymmetric symmetry, and as due to a dynamo if it has axisymmetry(Sofue et al. 1990). (It must be borne in mind that the actual magnetic field is perturbed by the spiral arms, so that it is parallel to the arms in the region of the spiral arms and toroidal in the region in between the arms as described in the introduction.) On the basis of our

analysis, we conclude that this method of distinguishing the origin is not valid, and could lead to incorrect conclusions concerning the origin of the galactic magnetic fields.

4. Parker(1968, 1973a,b) and others(Ruzmaikin et al 1988) have put forth three objections to a primordial origin. These objections are: (a) A primordial field would be tightly wrapped up, contrary to observations. (b) Such a field would be expelled by ambipolar diffusion or turbulent diffusion. (c) There is no known mechanism to produce a large scale primordial field in the early universe.

On the basis of our model we can counter the first two objections (a) and (b). Our model does indeed lead to a tightly wrapped spiral magnetic field. However, when averaged over a sufficiently large scale, as is done automatically by observations, the resulting field should actually appear to be axisymmetric and azimuthal. This averaged field is, thus, not in disagreement with observations. In addition, if the large scale field is not uniform, then the field after saturation by ambipolar diffusion creates larger regions of one sign then of the other sign and the averaged field is not zero. Thus, objection (a) does not defeat our model for a field of primordial origin.

With respect to objection (b) our model predicts field lines which thread through the disk, entering on one edge and leaving on the other. Thus, it is impossible for a vertical ambipolar motion or turbulent mixing to expel lines of force. This result counters objection (b).

The third objection is still open to debate and we do not discuss it.

The two observations that should test our model are: The magnetic field in other galaxies should be observed to be toroidal in between the arms. A reanalysis of the pulsar rotation measures should fit our model without very large extra fluctuations. These fluctuations should be accounted for by the rapid reversals of the toroidal field predicted by our model.

To summarize, we have shown that a careful analysis of the evolution of a primordial field throws new light on the way one should view the primordial field hypothesis.

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A. Appendix A

In this appendix we give the derivation of the solution given in equations 28-30, to the differential equations (24), (25), and (27) of section 2. Let us drop the θ subscript in B_θ. Then the equations to be solved are

$$\begin{aligned} \frac{dB_r}{dt} &= -\frac{v_D}{D}B_r \\ \frac{dB}{dt} &= -\frac{v_D}{D}B - \Omega B_r \\ \frac{v_D}{D}v_D &= kB^2. \end{aligned} \tag{A1}$$

Define ξ by

$$\frac{d\xi}{dt} = \frac{v_D}{D} = kB^2. \tag{A2}$$

Then

$$\frac{dB_r}{d\xi} = -B_r, \tag{A3}$$

or

$$B_r = B_1 e^{-\xi}, \tag{A4}$$

and

$$\frac{dB}{d\xi} = -B - \frac{\Omega}{kB^2} = -B - \frac{C}{B^2}e^{-\xi}, \tag{A5}$$

where $C = \Omega B_1/k$ is a constant.

Multiplying by e^ξ, we get

$$\frac{d}{d\xi}(Be^\xi) = \frac{C}{B^2} = \frac{Ce^{2\xi}}{(Be^\xi)^2}. \tag{A6}$$

Now, B is negative. Let $y = -Be^\xi$. Then

$$\frac{dy}{d\xi} = \frac{Ce^{2\xi}}{y^2}. \tag{A7}$$

Integrating and assuming the initial value of y is small we get

$$\frac{y^3}{3} = \frac{C}{2}(e^{2\xi} - 1), \tag{A8}$$

or

$$y = (\frac{3C}{2})^{1/3}(e^{2\xi} - 1)^{1/3}. \tag{A9}$$

Now,

$$e^{2\xi}\frac{d\xi}{d\tau} = kB^2 e^{2\xi}. = ky^2 \tag{A10}$$

Next, let $e^{\xi} = \eta$. Then

$$\frac{d\eta}{dt} = \frac{2k}{C}(\frac{3C}{2})^{2/3}(\eta - 1)^{2/3}, \tag{A11}$$

or

$$\frac{d\eta}{(\eta - 1)^{2/3}} = 2k(\frac{3}{2}C)^{2/3}dt. \tag{A12}$$

Integrating this equation we have

$$(\eta - 1)^{1/3} = \frac{2}{3}(\frac{3}{2}C)^{2/3}t \tag{A13}$$

Now expanding this equation and making use of the definition of η we have

$$e^{2\xi} = 1 + \frac{2}{3}C^2k^3t^3t^3. \tag{A14}$$

The toroidal field $B = ye^{-\xi}$, and y is given by equation A9 so

$$B^2 = y^2e^{-2\xi} = (\frac{3C}{2})^{2/3}\frac{(e^{2\xi} - 1)^{2/3}}{e^{2\xi}}, \tag{A15}$$

or

$$B^2 = \frac{C^2k^2t^2}{1 + (2/3)C^2k^3t^3}. \tag{A16}$$

Restoring the definition C, and setting $v_{D1} = kB_1^2$ we get equation (29) of section II for B_θ. Equation (28) for B_r is obtained from equation A4 and equation A14.

B. Appendix B

In this appendix we give the full equations for $dB'_r/dt, dB'_\theta/dt$ and dB'_z/dt including all the terms in R/D. They result from the transformation of equations 64 to 69 to dimensionless variables by making use of equations 70 to 77.

For convenience, the terms are listed in exactly the same order as in the original dimensional equations. The resulting equations are

$$\begin{aligned}\frac{\partial B'_r}{\partial t'} \;=\;& \frac{B'_\theta}{r'}\frac{\partial w'_r}{\partial z'} - B'_z\frac{\partial w'_r}{\partial z'} \\ & -\left(\frac{D}{R}\right) w'_r\frac{\partial B'_r}{\partial r'} - \left(\frac{D}{R}\right)^2 \frac{w'_\theta}{r'}\frac{\partial B'_r}{\partial u} - t'\frac{w'_r}{r'^2}\frac{\partial B'_r}{\partial u} - w'_z\frac{\partial B'_r}{\partial z'} \\ & -\left(\frac{D}{R}\right)\frac{w'_r}{r'}B'_r - \left(\frac{D}{R}\right)^2 \frac{B'_r}{r'}\frac{\partial w'_\theta}{\partial u} - B'_r\frac{\partial w'_z}{\partial z'}\end{aligned} \tag{B1}$$

$$\begin{aligned}\frac{\partial B_\theta}{\partial t'} \;=\;& = \left(\frac{D}{R}\right)^3 B'_r\frac{\partial w_\theta}{\partial r'} + \left(\frac{D}{R}\right)^2 \frac{B'_r}{r'^2}\frac{\partial w'_\theta}{\partial u}t' \\ & + \left(\frac{D}{R}\right) B'_z\frac{\partial w'_\theta}{\partial z'} - \left(\frac{D}{R}\right) w'_r\frac{\partial B_\theta}{\partial r'} - \left(\frac{D}{R}\right)^2 \frac{w'_\theta}{r'}\frac{\partial B'_\theta}{\partial u} - \frac{t'}{r'^2}w'_r\frac{\partial B'_\theta}{\partial u}\frac{\partial B_\theta}{\partial r'} \\ & - w'_z\frac{\partial B'_\theta}{\partial z'} - \frac{B'_r}{r'} \\ & - \left(\frac{D}{R}\right) B'_\theta\frac{\partial w'_r}{\partial r'} - \frac{t'}{r'^2}B'_\theta\frac{\partial w'_r}{\partial r'} - B'_\theta\frac{\partial w'_z}{\partial z'}\end{aligned} \tag{B2}$$

$$\begin{aligned}\frac{\partial B'_z}{\partial t'} \;=\;& \left(\frac{D}{R}\right) B'_r\frac{\partial w'_z}{\partial r'} + \frac{B'_\theta}{r'}\frac{\partial w'_z}{\partial u} + \frac{t'}{r'^2}B'_r\frac{\partial w'_z}{\partial u} \\ & - \left(\frac{D}{R}\right) w'_r\frac{\partial B'_z}{\partial r'} - \left(\frac{D}{R}\right)^2 \frac{w'_\theta}{r'}\frac{\partial B'_z}{\partial u} - \frac{t'}{r'^2}w'_r\frac{\partial B'_z}{\partial u} \\ & - w'_z\frac{\partial B'_z}{\partial z'} - \left(\frac{D}{R}\right) B'_z\frac{\partial w'_r}{\partial r'} \\ & - \frac{t'}{r'^2}B'_z\frac{\partial w'_r}{\partial u} - \left(\frac{D}{R}\right)\frac{B'_z}{r'}w'_r - \left(\frac{D}{R}\right)^2 \frac{B'_z}{r'}\frac{\partial w'_\theta}{\partial u}\end{aligned} \tag{B3}$$

The equations for the components of $\mathbf{w}$ are:

$$\begin{aligned} w'_r &= -\frac{t'}{r'^2}\frac{\partial B'^2}{\partial u} + \frac{\partial B'}{\partial r'} \\ w'_\theta &= -\frac{1}{r'}\frac{\partial B'^2}{\partial u} \\ w'_z &= -\frac{\partial B'^2}{\partial z'} \end{aligned} \tag{B4}$$

In these equations B'^2 stands for

$$B'^2 = B'^2_\theta + \left(\frac{D}{R}\right)^2 (B'^2_r + B'^2_z) \tag{B5}$$

We do not expand the expression for the components of $\mathbf{w}'$ out fully.

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FIGURE CAPTIONS

Fig. 1 (a) The protogalaxy before initial collapse, showing a uniform cosmic field threading it. The magnetic field makes an angle α with the rotation axis Ω. (b) The magnetic field after the protogalaxy has collapsed to the present size of the galaxy but before the galactic disk has formed. The lines remain threaded through the galactic plasma and connected to the rest of the cosmos. (c) The magnetic field lines after collapse to the galactic disk. Some of the lines such as a and d will escape by ambipolar diffusion and turbulent mixing. The lines b and c cannot escape. (d) The vertical ambipolar diffusion velocities act in opposite directions on a line of force to shorten its horizontal extent in the disk. They cannot remove it from the disk.

Fig. 2 The variation of the magnetic field strength of a line of force for different initial conditions. The field strength at first grows by stretching, and later decreases because of ambipolar diffusion. The saturated state of the lines is the same, independent of its initial field strength. The initial values of B in microgauss were, .01,.03, .1, .3, 1.0, and 3.

Fig. 3 The interstellar medium consists of dense clouds shown as spheres. A magnetic field line is shown which threads several clouds. The bowing up of the line in between the clouds is caused by cosmic ray and magnetic pressure in the intercloud medium. These pressures lead to a upward force on the neutrals in the cloud. The cloud is held down by gravitational force on the neutrals. The action of these two forces produces a motion of the ions through the cloud, ambipolar diffusion. The right hand end of the line is connected to the external cosmos. When the end of the line escapes from the last cloud by ambipolar diffusion, this part of the line will be expelled into the external world, and the part of the line in the disk will become shorter.

Fig. 4 The lines of force for a nonuniform magnetic field after compression into the disk. To preserve zero divergence the lines of force must fan out in the direction of weaker field. It is seen that the region of weaker field, where B_r is negative, is actually more extensive than

the region where B_r is positive. Thus, after the field everywhere reaches a saturated field strength this weaker region will lead to a region of larger area, and the final mean magnetic field will be dominated by it. (Because the saturated value is independent of the initial field strength, the weakness of the initial field will not be important, and only its more extensive area will be important.)

Fig.5 The variation of the magnetic field strength with z for different times as a result of the one dimensional calculation. At first the strength grows by stretching, and afterwards decreases by ambipolar diffusion. The value is negative because the initial radial field was positive, and the differential rotation sweeps it backwards. The magnetic field is plotted in dimensionless units. The dashed curves correspond to the times during which the field strength is increasing while the solid lines correspond to the decreasing phase. The convergence of the curves for the larger times is evident.

Fig. 6 The variation of the magnetic field with time for initial conditions $B_r = B_i \cos\theta$ for θ equal 0 to 180 degrees in 15 degree increments, where $B_i = .01$ in dimensionless units. These curves are derived from the one dimensional calculation. The convergence of the curves at large t parallels the same convergence in figure 2, and the curves for equal initial conditions agree quantitatively. (The values of the initial conditions in microgauss are $0.0285 \times \cos\theta + 0.000285 = 0.0288, 0.0250, 0.0205, 0.0146, 0.0077,$ etc.)

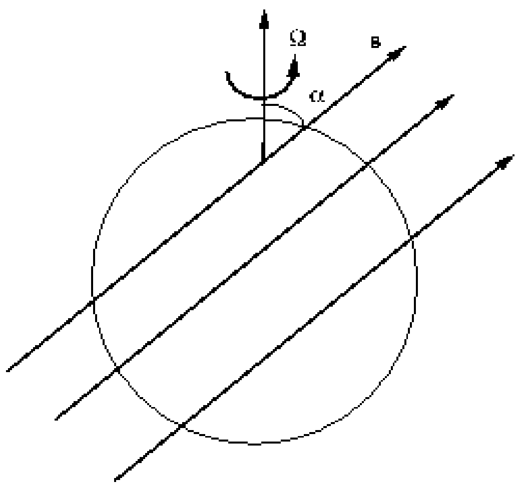
Fig. 7 The results of the two dimensional numerical simulation, for the case of an initial uniform magnetic field, B_θ is shown as function of u at $z = 0$ for different times. That the integral of B_θ is zero is evident. This field will not lead to any Faraday rotation. The labeling of the curves is the same as in figure 5.

Fig. 8 The results of the two dimensional numerical simulation, for the case of an initial non uniform magnetic field. B_θ is shown as function of u at $z = 0$ for different times. The regions of positive B_θ are broader than those of negative B_θ, so that the integral does not vanish, and this field *will* lead to a non zero Faraday rotation The labeling of the curves is the same

as in figure 5.

Fig. 9 The results of the two dimensional numerical simulation, for the case of an initial non uniform magnetic field. B_θ is shown as function of z at $u = 0$ for different times. The labeling of the curves is the same as in figure 5. The similarity to figure 5 is noteworthy.

Fig. 10 The results of the two dimensional numerical simulation, for the both cases. B_θ is shown as function of t at $u = 0, z = 0$. The behavior is quite similar to figure 2 for the one dimensional simulation.



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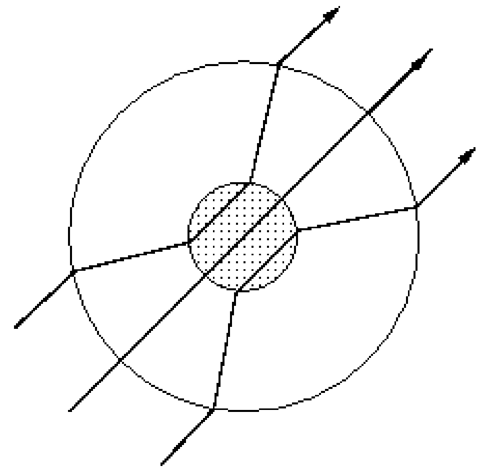

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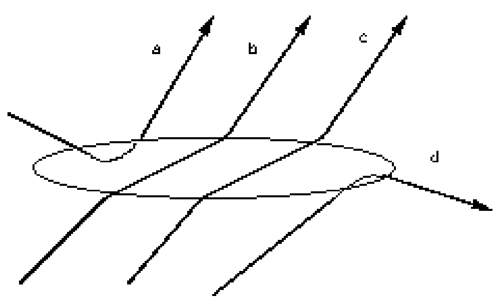

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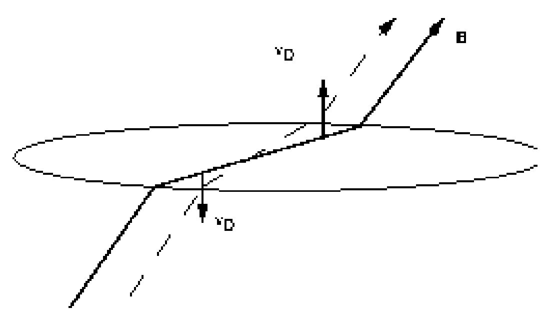

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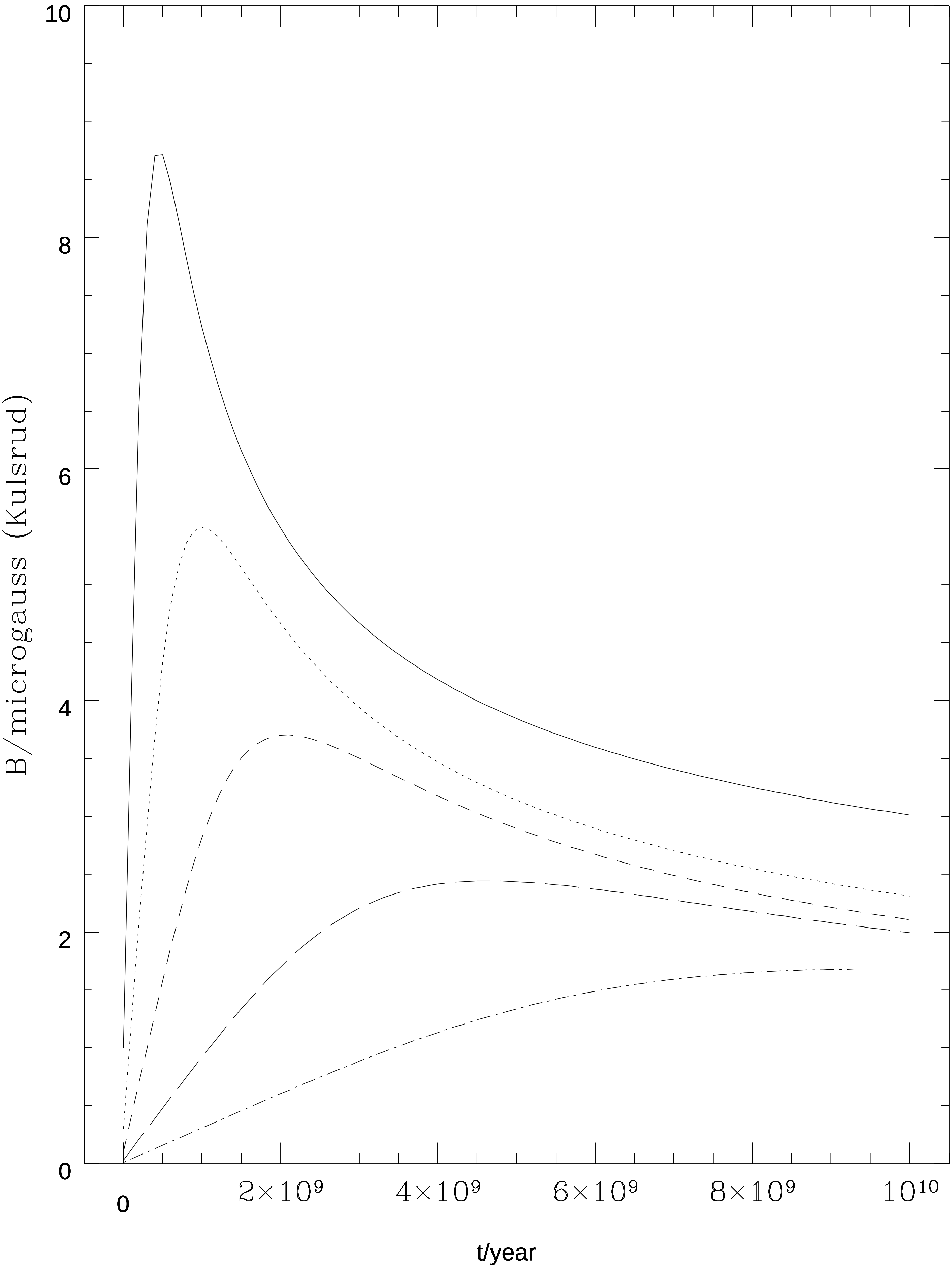
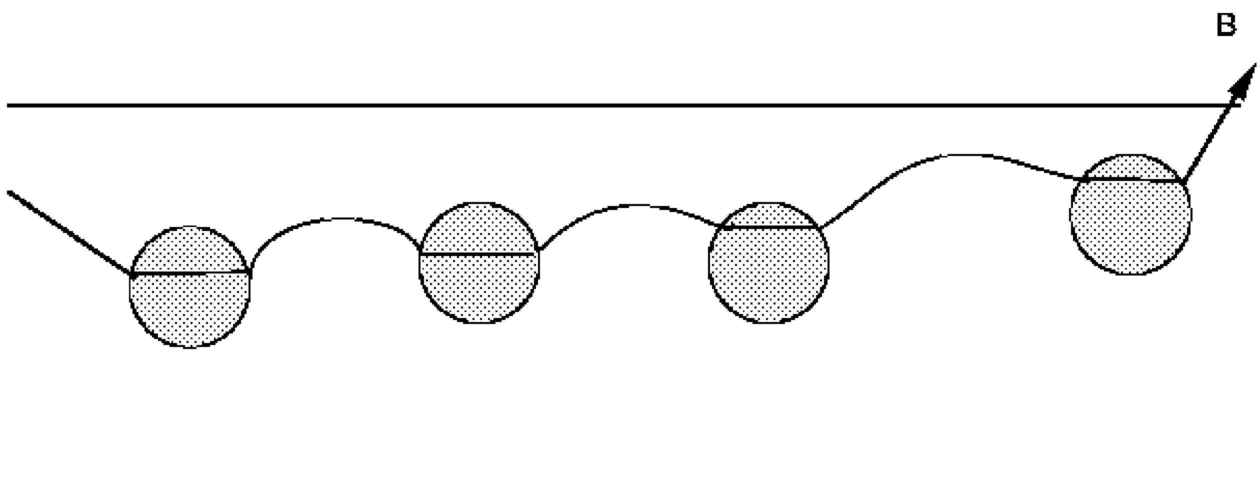
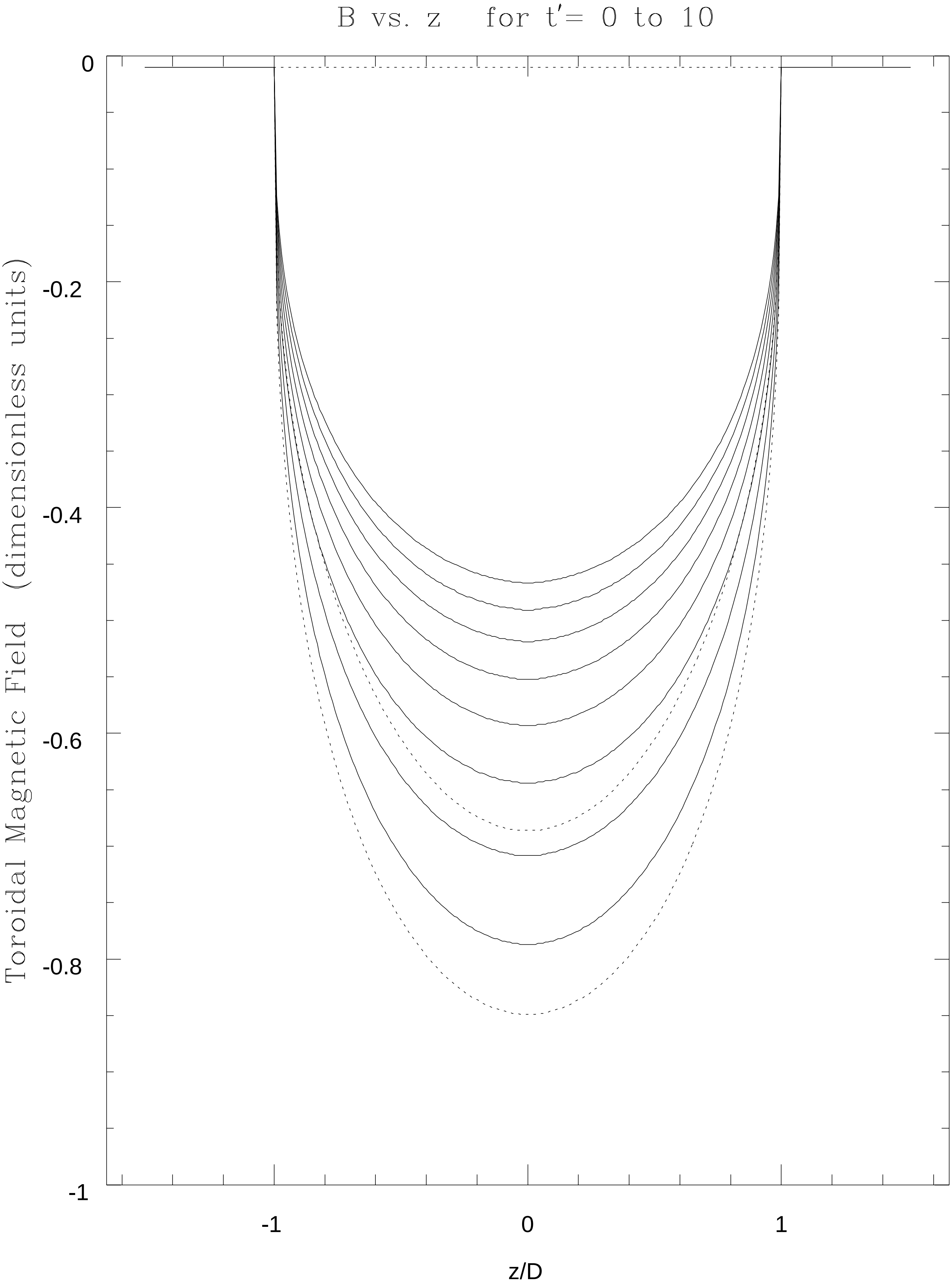
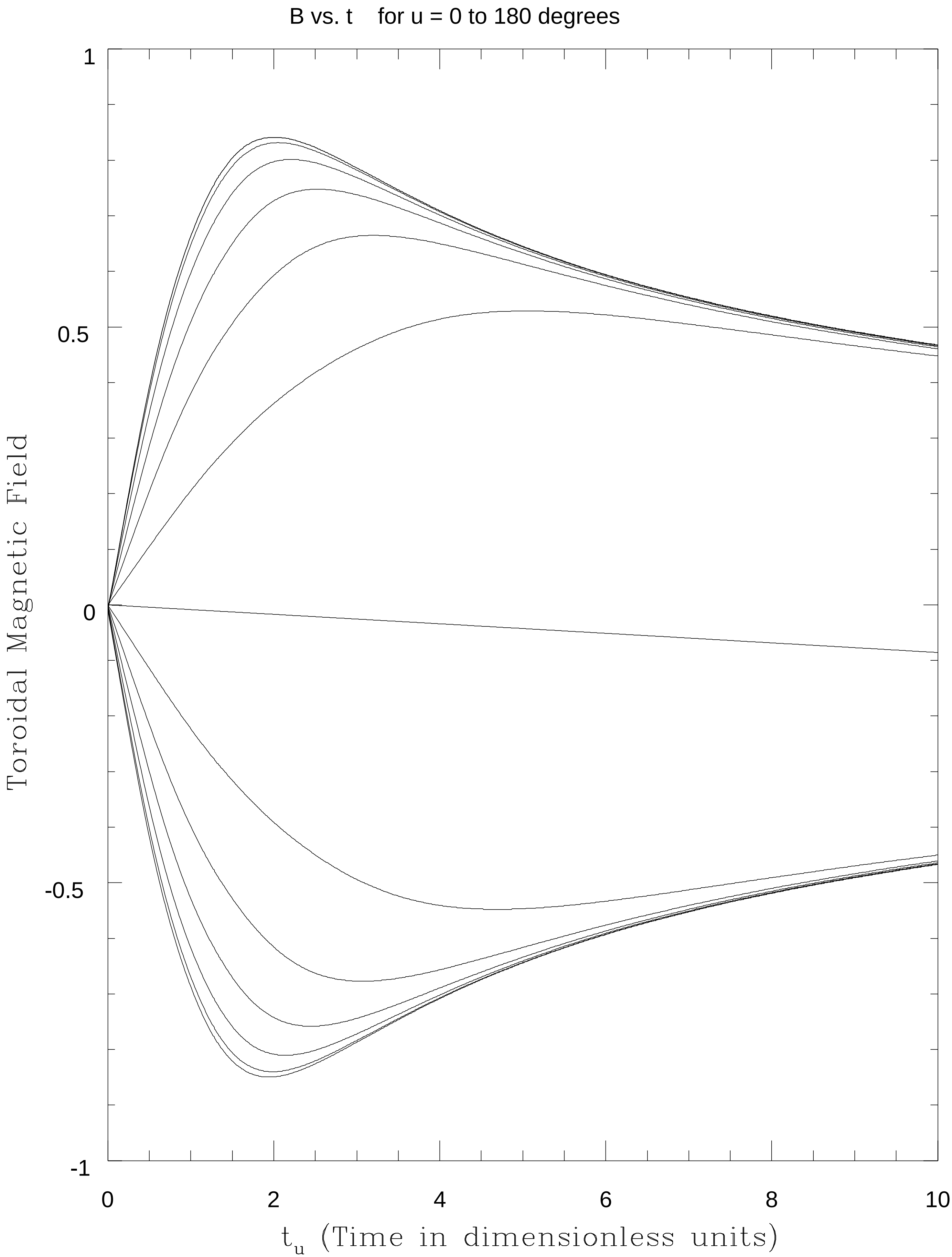
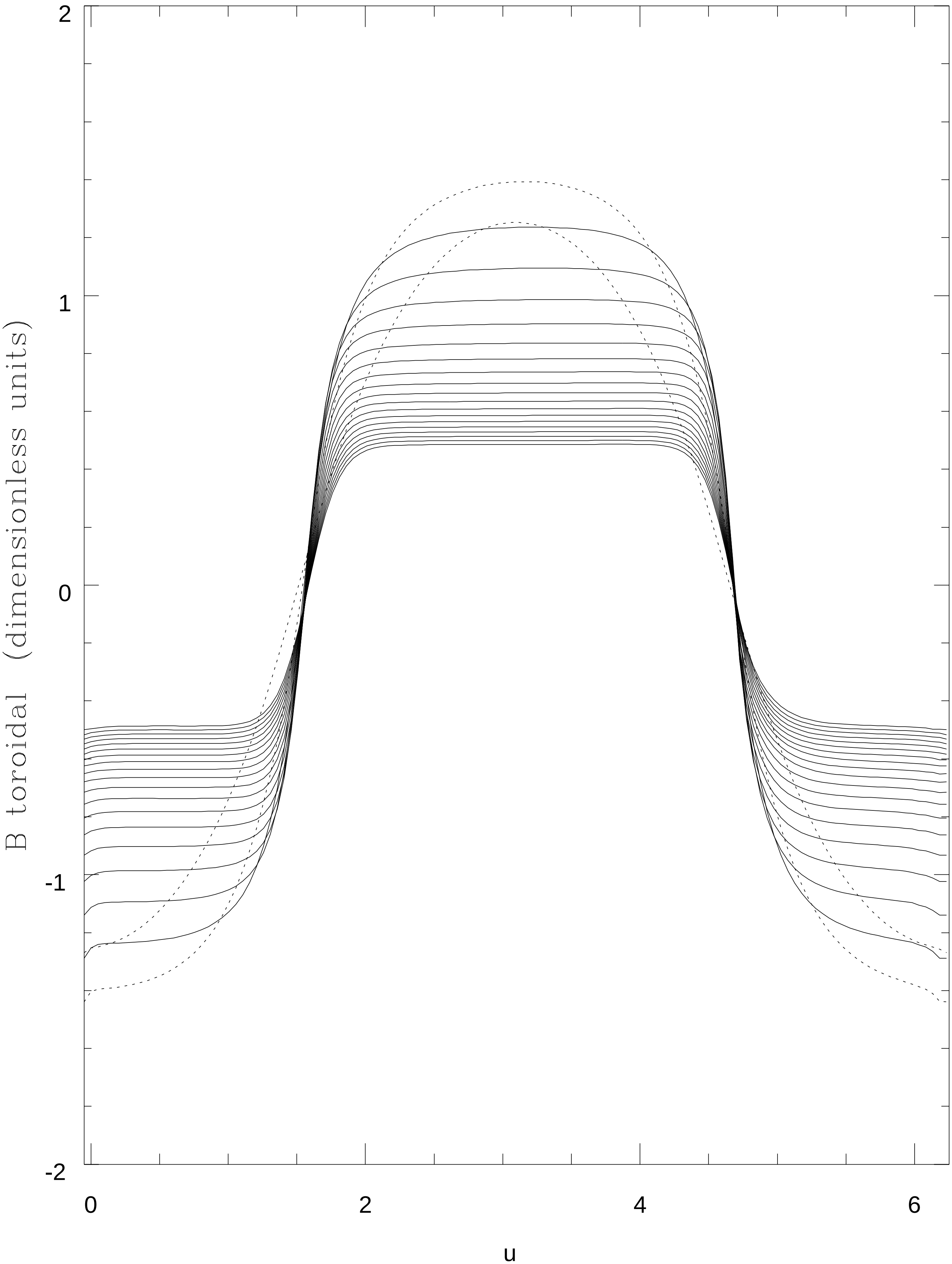


10
8
6
4
2
0
0
2×10⁹
4×10⁹
6×10⁹
8×10⁹
10¹⁰
B/microgauss (Kulsrud)
t/year

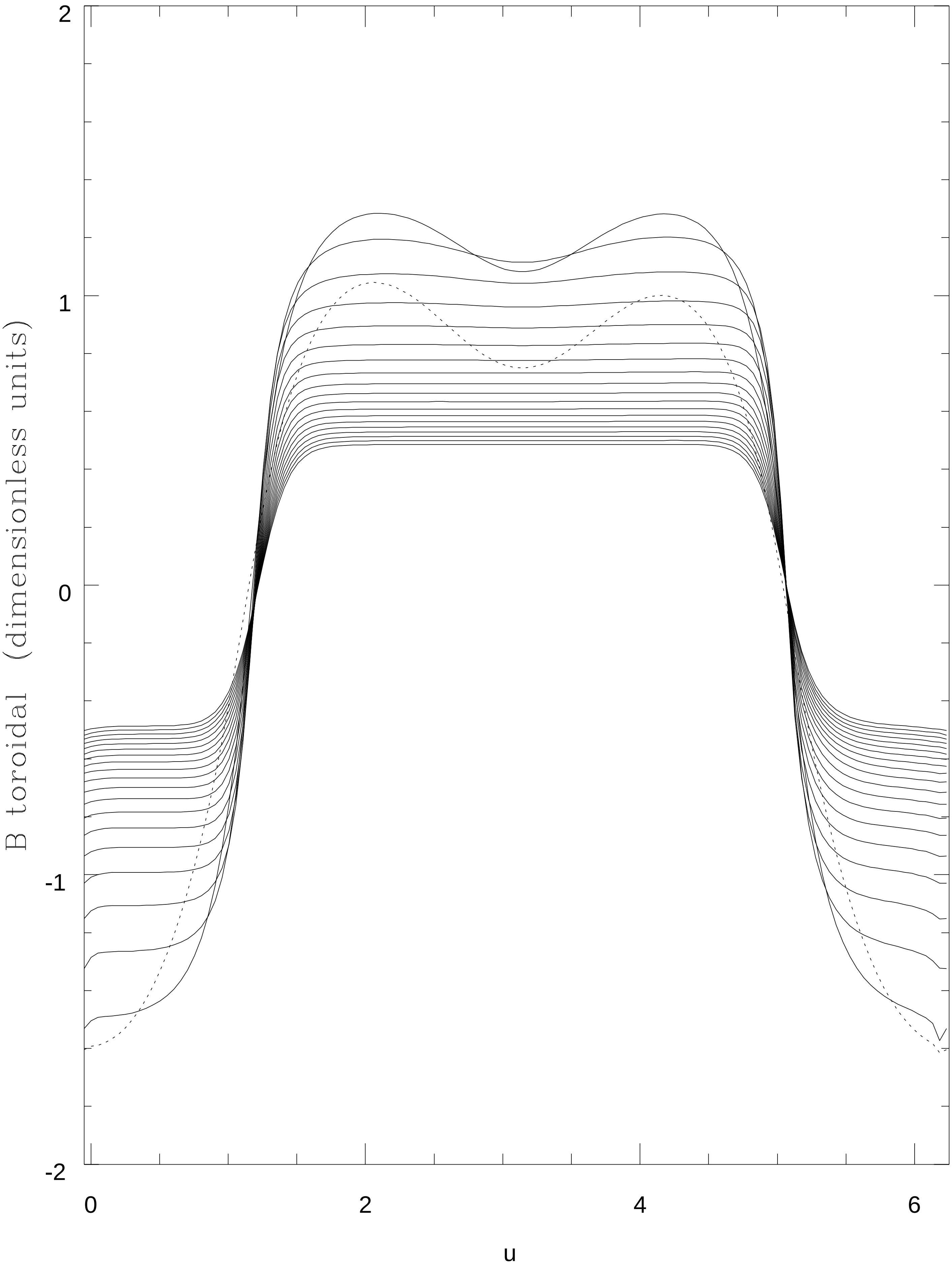

B


B vs. z for t′= 0 to 10
Toroidal Magnetic Field (dimensionless units)
z/D
0
-0.2
-0.4
-0.6
-0.8
-1
-1
0
1

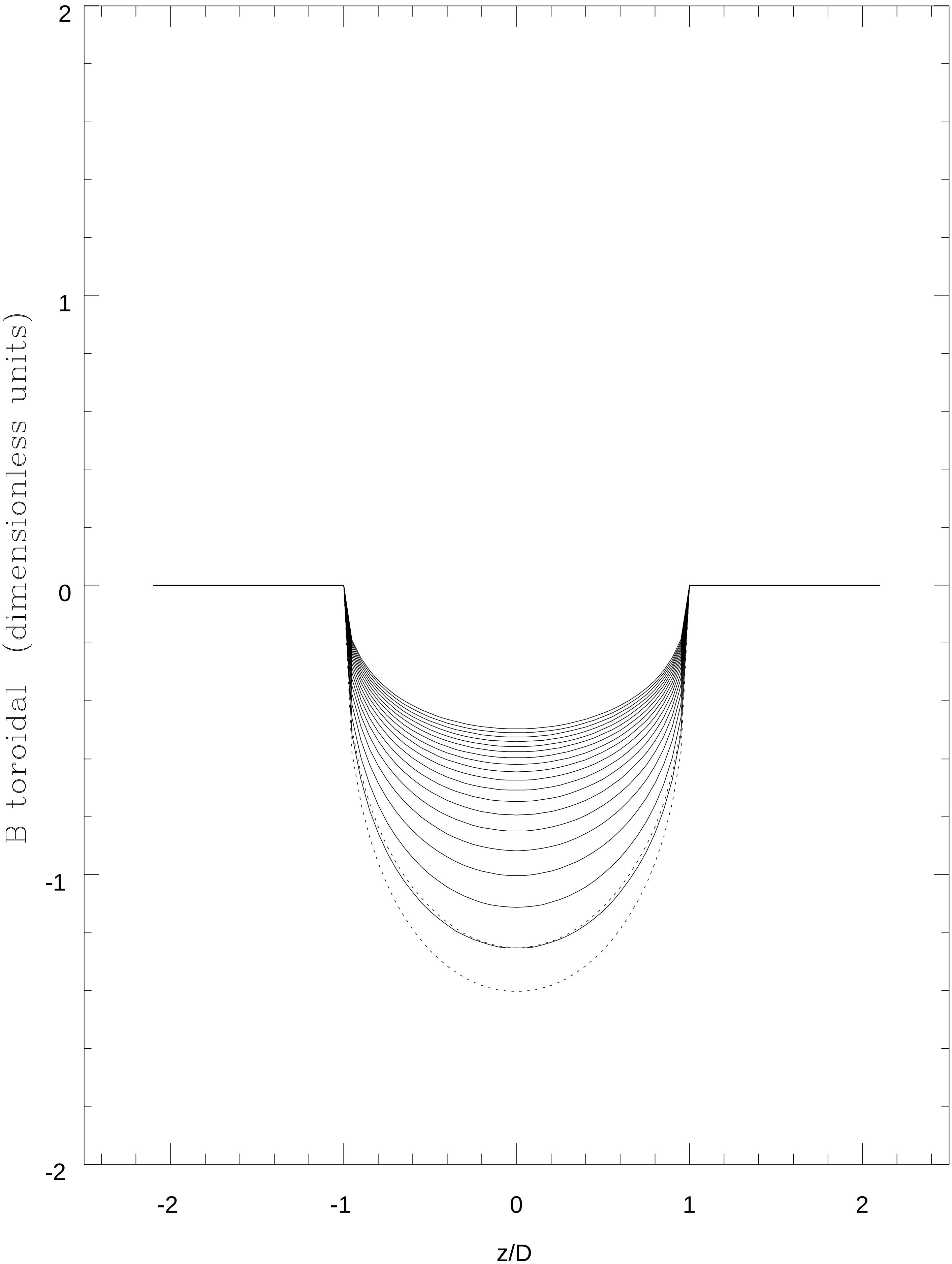

B vs. t for u = 0 to 180 degrees
Toroidal Magnetic Field
1
0.5
0
-0.5
-1
0
2
4
6
8
10
t_u (Time in dimensionless units)

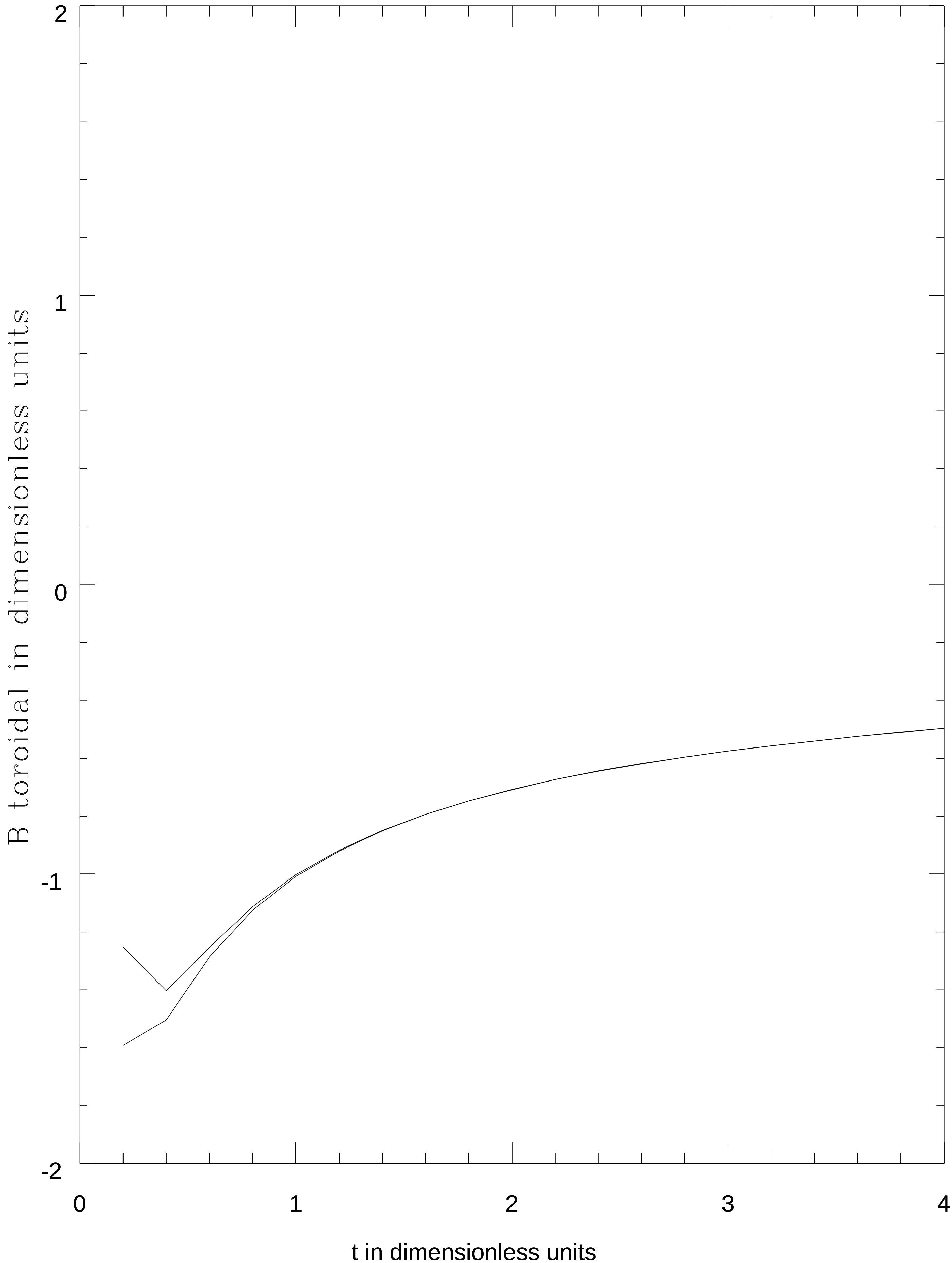


2
1
0
-1
-2
0
2
4
6
u
B toroidal (dimensionless units)



B toroidal (dimensionless units)
2
1
0
-1
-2
0
2
4
6
u


2
1
0
-1
-2
B toroidal (dimensionless units)
-2
-1
0
1
2
z/D


B toroidal in dimensionless units
2
1
0
-1
-2
0
1
2
3
4
t in dimensionless units